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LEVEL CROSSING PROBABILITIES FOR A MULTI-PARAMETER BROWNIAN PROCESS

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Let $\{\xi_{s_1,s_2}: -\infty < s_1, <\infty, -\infty < s_2 < \infty\}$ be a Gaussian process with $\xi_{s_1,s_2} = 0$ if $s_1 = 0$ or $s_2 = 0$, mean values $E(\xi_{s_1,s_2}) = 0$, and covariances $E(\xi_{s_1,s_2}\xi_{s_1',s_1'}) = 1/2 \min(s_1,s_1') \min(s_2,s_2')$. This is the two parameter Brownian process studied by J. D. Kuelbs, W. J. Park, P. T. Strait, and J. Yeh. In this paper, upper and lower bounds for level crossing probabilities of this process are derived.

More specifically, let (t_1, t_2) and (τ_1, τ_2) be two pairs of constants chosen so that $0 < t_1 < \infty, 0 < t_2 < \infty, 0 < \tau_1 < \infty$, and $0 < \tau_2 < \infty$. Let $\delta_1 = \tau_1/m$, $\delta_2 = \tau_2/n$ where m and n are integers, and define random variables $X_{h,k}$ for $h = 0, 1, 2, \cdots, m$ and $k = 0, 1, 2, \cdots, n$ as follows.

$$X_{h,k} = \begin{cases} 0 & \text{for} \quad h = 0 \quad \text{or} \quad k = 0 \\ \xi_{t_1+(h-1)\delta_1, t_2+(k-1)\delta_2} & \text{for} \quad h = 1, 2, \cdots, m; k = 1, 2, \cdots, n . \end{cases} \tag{1}$$

For any given number a, define

$$\begin{aligned} &P_{m,n}(a, t_1, t_2, \tau_1, \tau_2) \\ &\quad = P(X_{i,j} > a \text{ for } i = 1, 2, \cdots, m; j = 1, 2, \cdots, n) . \end{aligned} \tag{2}$$

In this paper, upper and lower bounds (Theorems 1 and 2) for $\lim_{m\to\infty\, n\to\infty} P_{m,n}(a, t_1, t_2, \tau_1, \tau_2)$ are derived.

2. Preliminary lemmas.

LEMMA 1. *Let $\zeta_{h,k} = X_{h,k} - X_{h,k-1} - X_{h-1,k} + X_{h-1,k-1}$ for $h = 1, \cdots, m$ and $k = 1, \cdots, n$. Then,*

(i) *$\zeta_{h,k}$, $h = 1, \cdots, m$, $k = 1, \cdots, n$ are independent Gaussian random variables with means 0 and variances $\sigma^2_{h,k}$ given by*

$$\begin{aligned} \sigma^2_{1,1} &= \frac{1}{2} t_1 t_2 \\ \sigma^2_{1,k} &= \frac{1}{2}\delta_1 t_2 \quad \text{for} \quad k = 2, \cdots, n \\ \sigma^2_{h,1} &= \frac{1}{2}\delta_2 t_1 \quad \text{for} \quad h = 2, \cdots, m \\ \sigma^2_{h,k} &= \frac{1}{2}\delta_1\delta_2 \quad \text{for} \quad h = 2, \cdots, m; k = 2, \cdots, m \end{aligned} \tag{3}$$