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# NONSINGULAR DEFORMATIONS OF A DETERMINANTAL SCHEME

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**We will be considering an affine algebraic scheme $X$ over a field $k$, which is determinantal, defined by the vanishing of the $l \times l$ minors of a matrix $R$.**

**We will show that deforming the constant and linear terms of the entries in the matrix $R$ gives an almost everywhere flat deformation of $X$, and that under certain simple conditions, and in particular if the dimension of $X$ is sufficiently low, this deformation has generically nonsingular fibers.**

Essentially the same results were obtained simultaneously by D. Laksov [3] using more general theorems on transversality of mappings. He quotes a result of T. Svanes indicating that the codimension result, identical in both versions, is the best obtainable (see Example 3).

This article is a generalization of an earlier result about nonsingular deformations of Cohen-Macaulay schemes of codimension 2 (Schaps [4]). Moreover, since determinantal schemes were introduced by Macaulay as a generalization of complete intersection, the theorem proven in this paper can be regarded as a generalization of Bertini's theorem, that the generic deformation of a complete intersection is nonsingular.

The precise definition of a determinantal scheme is as follows:

DEFINITION. An affine scheme $X = \operatorname{Spec}(k[Z_1, \cdots, Z_q]/J)$ is determinantal if $J$ is generated by all the $l \times l$ minors of an $m \times n$ matrix $R$ of polynomials, and $X$ is equidimensional of codimension $(m - l + 1)(n - l + 1)$.

On the course of the theorem, we will need to use the generic determinantal scheme, constructed as follows: Let $Y = (Y_{ij})$, $i = 1, \cdots, m$, $j = 1, \cdots n$, be a set of indeterminates, and let $P_l^Y$ be the ideal generated in $k[Y]$ by the $l \times l$ minors of the matrix $[Y_{ij}]$. Then it is known that $P_l^Y$ is a prime ideal of height $(m - l + 1)(n - l + 1)$. This number is thus the maximal codimension that can be obtained by a scheme generated by minors of this order in an $m \times n$ matrix. We will use a recent result by Hochster and Eagon [2], that every determinantal scheme is Cohen-Macaulay.