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GROUP REPRESENTATIONS ON HILBERT SPACES DEFINED IN TERMS OF $\bar{\partial}_b$-COHOMOLOGY ON THE SILOV BOUNDARY OF A SIEGEL DOMAIN

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Let Q be a C^n-valued quadratic form on C^m. Let $N(Q)$ be the 2-step nilpotent group defined on $R^n \times C^m$ by the group law

$$(x, u)\cdot(x', u') = (x + x' + 2\,\mathrm{Im}\,Q(u, u'), u + u') .$$

Then $N(Q)$ has a faithful representation as a group of complex affine transformations of C^{n+m} as follows:

$$g\cdot(z, u) = (z + x_0) + i(2Q(u, u_0) + Q(u, u_0), u_0 + u_0) ,$$

where $g = (x_0, u_0)$. The orbit of the origin is the surface

$$\Sigma = \{(z, u) \in C^{n+m}; \mathrm{Im}\, z = Q(u, u)\} .$$

This surface is of the type introduced in [11], and has an induced $\bar{\partial}_b$-complex (as described in that paper) which is, roughly speaking, the residual part (along Σ) of the $\bar{\partial}$-complex on C^{n+m}. Since the action of $N(Q)$ is complex analytic, it lifts to an action on the spaces E^q of this complex which commutes with $\bar{\partial}_b$. Since the action of $N(Q)$ is by translations, the ordinary Euclidean inner product on C^{n+m} is $N(Q)$-invariant, and thus $N(Q)$ acts unitarily in the L^2-metrics on $C_0^\infty(E^q)$ defined by

$$\|\Sigma a_I d\bar{u}_I\|^2 = \int_\Sigma \Sigma |a_I|^2 dV$$

where dV is ordinary Lebesgue surface measure. In this way we obtain unitary representations ρ_q of $N(Q)$ on the square-integrable cohomology spaces $H^q(E)$ of the induced $\bar{\partial}_b$-complex.

These are generalizations of the so-called Fock or Segal-Bargmann representations [2, 4, 10, 13], and the representations studied by Carmona [3]. In this paper, we explicitly determine these representations and exhibit operators which intertwine the ρ_q with certain direct integrals of the Fock representations.

This is accomplished by means of a generalized Paley-Wiener theorem arising out of Fourier-Laplace transformation in the x ($\mathrm{Re}\, z$) variable. Let us describe this result. For $\xi \in R^{n*}$, let $Q_\xi(u, v) = \langle \xi, Q(u, v)\rangle$. Let $H^q(\xi)$ be the square-integrable cohomology of the $\bar{\partial}$-complex on C^m relative to the norm