PACIFIC JOURNAL OF MATHEMATICS
Vol. 65, No. 1, 1976

CLOSED ORIENTED 3-MANIFOLDS AS 3-FOLD BRANCHED COVERINGS OF S^3 OF SPECIAL TYPE

HUGH M. HILDEN, JOSE M. MONTESINOS AND THOMAS THICKSTUN

It has been shown by Hilden and Montesinos independently that any closed oriented 3-manifold M is a 3-fold irregular branched covering of S^3, $p: M \to S^3$. The purpose of this paper is to show that the branch covering space map can be chosen in such a way that the set of points at which p fails to be a local homeomorphism is the boundary of a disc in M. One application of this result is a new proof that a closed oriented 3-manifold is parallelizable.

1. Introduction. Let $p: X \to Y$ be a nondegenerate simplical map between compact triangulated combinatorial n-manifolds such that $p(\partial X) \subset \partial Y$. If the restriction of p to a map of the complements of the $n-2$ skeletons is a covering space map, then $p: X \to Y$ is called a *branched covering space*. A point $x \in X$ is called an ordinary point if p maps some neighborhood of x homeomorphically into Y. The complement of the set of ordinary points is called the *branch cover*. The image of the branch cover is called the *branch set*.

A *represented link* is a polygonal link L in S^3 together with a transitive representation ω of the link group $\pi(S^3 - L)$ into Σ_d, the symmetric group on d-symbols. The representation is called *simple* if each meridian of L is represented by a transposition.

Given a represented link (L, ω), there is a unique closed orientable 3-manifold, $M(L, \omega)$, determined by the representation. The manifold $M(L, \omega)$ is a d-fold branched covering space of S^3 branched over L. A detailed description of how to construct $M(L, \omega)$ is given in [9].

If M is a compact orientable 3-manifold and $p: M \to S^3$ is a branched covering space such that the branch set is a link L, then $M = M(L, \omega)$ where $\omega(\gamma)$ is the permutation γ induces on the left cosets of $p_*\pi_1(M - p^{-1}(L))$ by left multiplication.

Alexander's theorem ([1]) states that any closed orientable 3-manifold is an $M(L, \omega)$ for some link L and some presentation ω. Two of the co-authors of this paper showed, in [6], [7], [4] and [5], using different methods, that any closed orientable 3-manifold is an $M(L, \omega)$, where ω is a *simple* representation of $\pi(S^3 - L)$ onto Σ_3 and L is a knot. Thus any closed orientable 3-manifold is an irregular