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CHARACTERIZATIONS OF SOME C^*-EMBEDDED SUBSPACES OF $\beta\underline{N}$

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Let K be a compact F-space such that $|C^*(K)| = 2^\omega$. Using the continuum hypothesis we characterize those subspaces of K that are C^*-embedded in K. We also characterize the class of extremally disconnected Tychonoff spaces of countable cellularity. As corollaries of these theorems, using various set-theoretic hypotheses we characterize the C^*-embedded, and the extremally disconnected C^*-embedded, subspaces of $\beta\underline{N}$.

1. **Introdution.** Our notation and terminology follows that of the Gillman-Jerison text [4]. All hypothesized topological spaces are assumed to be completely regular and Hausdorff (i.e., Tychonoff). As usual βX denotes the Stone-Čech compactification of the Tychonoff space X, and $\underline{N}$ denotes the countable discrete space. $C^*(X)$ denotes the family of bounded real-valued continuous functions on X. A subspace S of X is *C^*-embedded* in X if given $f \in C^*(S)$ there exists $g \in C^*(X)$ such that $g|S = f$. A *cozero-set* of X is a set of the form $X - f^{\leftarrow}(0)$ where $f \in C^*(X)$. The collection of cozero-sets of X is denoted by $\operatorname{coz}(X)$. A space X is *zero-dimensional* if its open-and-closed (clopen) sets form a base for its open sets. X is *strongly zero-dimensional* if βX is zero-dimensional.

A space X is *weakly Lindelöf* if given an open cover $\mathscr{V}$ of X, there is a countable subfamily $\mathscr{C}$ of $\mathscr{V}$ such that $\bigcup \mathscr{C}$ is dense in X (if $\mathscr{C}$ is a collection of subsets of a set we denote $\bigcup\{C: C \in \mathscr{C}\}$ by $\bigcup \mathscr{C}$). A space X has the *countable chain condition*, or *countable cellularity*, if each family of pairwise disjoint nonempty open subsets of X is countable. We abbreviate this by writing "X has c.c.c." The following lemma, which came to the attention of the author through a letter from W.W. Comfort, is easily proved.

LEMMA 1.1. *A space has* c.c.c. *iff each of its open subsets is weakly Lindelöf.*

A space X is *extremally disconnected* if disjoint open subsets have disjoint closures. It is an *F-space* if its cozero-sets are C^*-embedded. It is an *F'-space* if disjoint cozero-sets have disjoint closures. Each extremally disconnected space is an F-space, and each F-space is an F'-space. Proofs of these facts, plus other information on these classes of spaces, may be found in [1] and [4]. We shall