PACIFIC JOURNAL OF MATHEMATICS
Vol. 65, No. 2, 1976

THE STRUCTURE OF SUBLATTICES OF THE PRODUCT OF n LATTICES

DONALD M. TOPKIS

The structure of sublattices of the product of n lattices is explored. Such a sublattice is decomposed and completely characterized in terms of $n(n-1)/2$ sublattices of the product of two lattices. A sublattice of the product of two lattices is represented in terms of several easily characterized sublattices. The sublattice characterizations provide analogous characterizations for those functions whose level sets are sublattices. A simple representation is also given for the sections of a sublattice of the product of two lattices.

Introduction. I will proceed to explore the structure of sublattices of the product of n lattices. It will be shown that such general sublattices can be represented in terms of some other sublattices which are quite simple to conceptualize and characterize.

The results on sublattice structure are given in § 1. In Theorem 1 a sublattice of the product of n lattices is decomposed so that it is completely characterized in terms of $n(n-1)/2$ sublattices of the product of two lattices. Lemma 2 and Corollary 1 give simple representations for sections of a sublattice of the product of two lattices. Theorem 2 represents sublattices of the product of two lattices by several easily characterized sublattices of this product. Theorem 3 combines previous results to provide a more refined characterization of sublattices of the product of n lattices.

Often sets are constructed as the intersection of level sets of a system of functions. For instance, this is frequently the case in defining the feasible region for optimization problems. To recognize when such sets are sublattices one must know what functions have sublattices as their level sets. Thus in § 2 the results of § 1 are translated into analogous characterizations for those functions whose level sets are sublattices.

These results are handy in dealing with structured optimization problems considered by the author [5, 6, 7, 9, 10, 11]. In [5, 9] a theory is developed for certain structured optimization problems in which each constraint set must be a sublattice. In order to recognize and generate domains which are sublattices (so the theory may be applied) as well as to envision the possible range of applicability of