

A CLASS OF T_1 -COMPACTIFICATIONS

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In this work the correspondence between T_2 -compactifications, proximity relations, and families of maximal round filters is extended to the case of T_1 -spaces. The major results spell out bijections between a special class of T_1 -compactifications, certain proximity relations on the original space, and certain filterfamilies. Perhaps the most interesting result is the identification of a class of compactifications between T_1 and T_2 -compactifications. This class consists of the principal weakly regular minimal compactifications and includes the Wallman compactification, and also the one-point compactification of a locally compact space. Moreover, the Wallman compactification is the largest weakly regular minimal compactification of a T_1 -space. This improves the known result that the Wallman compactification is T_1 , and is larger than any T_2 -compactification.

1. Extension structures and T_1 -compactifications. This section develops the notion of a compactification structure, which is a family of filters satisfying conditions strong enough to guarantee that the induced extension is a compactification. These induced compactifications constitute a class lying between T_1 and T_2 -compactifications. In the language developed in this section, it is the class of principal weakly regular minimal compactifications. The main result is the 1-1 correspondence between these compactifications and the set of compactification structures on a given T_1 -space. The Wallman compactification is characterized as the largest weakly regular minimal compactification of a T_1 -space. Much of the notation and terminology in this section is taken from Thron [4], Chapter 17.

DEFINITION 1.1. An *extension structure* on a topological space (X, \mathcal{S}) is a family Φ of open filters on X which includes all neighborhood filters. We will call the extension structure T_1 if no filter in Φ contains any other filter in Φ .

The inverse images of the neighborhood filters of an extension constitute an extension structure. We will call this the *trace system* of the extension. Conversely, each extension structure is the trace system of some extension.

This correspondence between extensions and extension structures is not 1-1, since inequivalent extensions can have the same trace system. However, for each extension structure Φ there is a preferred extension $\pi(\Phi)$, which we will call the *principal extension* associated