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# THE ABSOLUTE BAIRE PROPERTY

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**In earlier papers the author has formulated an axiomatic foundation for a general theory of point sets, one of whose purposes is the unification of analogous theorems concerning Baire category and Lebesgue measure. Within this context, a method is given in the present paper for unifying the Baire property in the restricted sense and absolute measurability on the real line.**

**That absolute measurability is the appropriate measure-theoretic analogue of the Baire property in the restricted sense was suggested by E. Marczewski in a classical paper in 1937, after he had established the Baire property (in the wide sense) and Lebesgue measurability as analogous concepts.**

Except for §§1 and 5, $X$ will denote the real line. In §1 a brief review is given of basic definitions and facts from [6] which are pertinent to this paper. We define in §2 the "absolute Baire property" in terms of order preserving mappings. That this definition actually effects the desired unification is dependent upon the intimate relationship existing between perfect sets and sets of order type $\lambda$ (the order type of the real line) as discussed in §3. The classical examples are then presented in §4.

A central role in these investigations is played by certain families of perfect sets whose properties are given in §§5 and 6. In §7 we prove a general theorem which includes as special cases the known invariance under order isomorphisms of absolute measurability and of Marczewski's sets. Finally, some open problems are stated in §8.

## 1. Preliminaries.

NOTATION. If $\mathscr{S}$ is any family of sets, then the members of $\mathscr{S}$ will be called $\mathscr{S}$-sets.

Upon isolating properties common to the families of all closed intervals, all perfect sets, and all closed sets of positive Lebesgue measure, the following notion of a $\mathfrak{K}$-family was obtained (see [6]).

DEFINITION 1. A pair $(X, \mathscr{C})$ where $X$ is a nonempty set and $\mathscr{C}$ is a family of subsets of $X$ is a $\mathfrak{K}$-family if the following axioms are satisfied.

1. $X = \bigcup \mathscr{C}$.

2. Let $A$ be a $\mathscr{C}$-set and let $\mathscr{D}$ be a nonempty family of disjoint $\mathscr{C}$-sets which has power less than the power of $\mathscr{C}$.