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MAXIMAL IDEALS IN ALGEBRAS OF TOPOLOGICAL ALGEBRA VALUED FUNCTIONS

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For a completely regular space T, topological algegbra A and algebra X, both commutative and having identity, let $C(T, A)=\{f: T \rightarrow A: f$ is continous$\}$, $C^*(T, A)=\{f \in C(T, A): f(T)$ is relatively compact$\}$ and $\mathscr{M}(X)$ be the set of all maximal ideals of codimension one in X endowed with the Gelfand topology (i.e., the weak topology generated by $\{\hat{x}: x \in X\}$, where $\hat{x}(m)=x+m$). When A is the real numbers, the spaces $\mathscr{M}(C(T, A))$ $(=\upsilon T)$ and $\mathscr{M}(C^*(T, A))$ $(=\beta T)$ are well known. If A is any topological algebra, $t \in T$ and $m \in \mathscr{M}(A)$, then $M_{t,m}=\{f \in C(T, A): f(t) \in m\} \in \mathscr{M}(C(T, A))$, and $(t, m) \rightarrow M_{t,m}$ is an injection of $T \times \mathscr{M}(A)$ into $\mathscr{M}(C(T, A))$. It is shown that if T is realcompact, A is a Q algebra with continuous inversion and either $\mathscr{M}(A)$ is locally equicontinuous or T is discrete, then this injection is a homeomorphism. It is further shown that if the assumption about T is reduced to complete regularity, then $\mathscr{M}(C^*(T, A))$ is homeomorphic to $(\beta T) \times \mathscr{M}(A)$, and if A is also realcompact, then $\mathscr{M}(C(T, A))$ is homeomorphic to $(\upsilon T) \times \mathscr{M}(A)$. These results are obtained for topological algebras over the reals, the complexes and certain ultraregular topological fields (including all non-archimedean valued fields) with no assumptions of local convexity.

1. We assume that the reader is familiar with the properties of $C(T, A)$ and $C^*(T, A)$ for T completely regular and A the real or complex numbers, as presented in Gillman and Jerison [4]. For a development of analogous results when A is an ultraregular topological field (an ultraregular space is one whose topology has a base of sets which are both open and closed), the reader is referred to Bachman, Beckenstein, Narici and Warner [1]. In this case, T is also assumed to be ultraregular, the Banaschewski compactification $(\beta_0 T)$ is analogous to the Stone-Cech compactification (βT), F-replete is analogous to realcompact and the F-repletion $(\upsilon_F T)$ is analogous to the realcompactification (υT). Except where noted, all pairs (T, A) used below are assumed to satisfy either of two sets of conditions: T is completely regular and A is a commutative topological algebra with identity e over the real or complex numbers, or T is ultraregular, A is a commutative topological algebra with identity e over a complete ultraregular topological field F, and disjoint F-zero sets in T (i.e., inverse images of $\{0\}$ under continuous functions from T into F) have disjoint closures in $\beta_0 T$ (which will hold if the field is met-