

GEOMETRY AND THE RADON-NIKODYM THEOREM IN STRICT MACKAY CONVERGENCE SPACES

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The main purpose of this paper is to indicate a technique for extending certain types of results which are known for Banach spaces to the corresponding results in more general locally convex topological vector spaces. We shall extend these results to the class of locally convex spaces possessing the strict Mackey convergence [SMC] property. The technique involves a natural embedding into a Banach space and an application of the Banach space result. Loosely speaking, we have that those properties of closed bounded sets in Banach spaces that do not depend upon any open set will yield analogous results for closed bounded sets in quasicomplete locally convex spaces with the SMC property.

Peck [16] and Saab [20] have used a constructive embedding technique for extending results to Fréchet spaces and we shall see that for spaces with the SMC property the embedding used is obtained in a very natural manner without necessity of construction.

In the first section we shall be concerned with vector integration, the Radon-Nikodym Theorem [RNT], and the Radon-Nikodym Property [RNP] for spaces with the SMC property. Rieffel [18, 19], Maynard [13], and others [1, 5] have proved a RNT for the Bochner and the Dunford second integral in Banach spaces. By modifying their techniques, Chi [3] has extended this theorem to Fréchet spaces. We shall extend the Dunford second integral to spaces with the SMC property and obtain the RNT directly from the Banach space result.

Banach spaces with the RNP have been characterized [5, 9, 10, 13, 17] in terms of the geometric and extremal structure of their closed bounded convex sets. Saab [20] and Chi [3] have extended some of these results to Fréchet spaces for a Bochner type integral. We shall obtain these results for our integration theory in spaces with the SMC property.

The second section deals with the separation of closed bounded convex sets, denseness of support points for closed bounded convex sets and denseness of support functionals for closed bounded convex sets. We shall give two results on weakly compact subsets of spaces with the SMC property. Bishop and Phelps [1] gave several results on support points of convex sets in Banach spaces and Peck [16] has given analogs of some of them in Fréchet spaces. We