PACIFIC JOURNAL OF MATHEMATICS
Vol. 65, No. 2, 1976

AN APPROXIMATION THEOREM FOR MAPS INTO KAN FIBRATIONS

RUDOLF FRITSCH

In this note we prove that a semisimplicial map into the base of a Kan fibration having a continuous lifting to the total space also has a semisimplicial lifiting, very "close" to a given continuous lifting. As a special case we obtain a new proof of the famous Milnor-Lamotke theorem that a Kan set is a strong deformation retract of the singular set of its geometric realization.

First we state our main

THEOREM. *Let*

$$(*)\qquad \begin{array}{ccc} X & \xrightarrow{f} & E \\ {\scriptstyle i}\downarrow & & \downarrow{\scriptstyle p} \\ Y & \xrightarrow[h]{} & B \end{array}$$

be a commutative square in the category of semisimplicial sets with i *an inclusion and* p *a Kan fibration. Further, suppose given a continuous* $\bar{g}: |Y| \to |E|$ *with* $\bar{g} \circ |i| = |f|$ *and* $|p| \circ \bar{g} = |h|$. *Then there exists a homotopy* $\bar{g} \cong g'$ *rel.* $|X|$ *and over* $|B|$ *so that* $g' = |g|$ *for some semisimplicial* g.

This theorem has an interesting special case. Take $X = E$ a Kan set, $Y = S|E|$, B a point, p, h the unique constant maps, $f = idE$, i the natural inclusion and $\bar{g}$ the natural retraction. What comes out is the famous Milnor-Lamotke theorem saying E is strong deformation retract of $S|E|$. Thus we get a new proof of this theorem which in contrast to the original one [4] avoids any reference to J.H.C. Whitehead's theorems.

On the other hand, if B is a point, the statement is a trivial consequence of the Milnor-Lamotke theorem. An elementary proof for this case—avoiding the Milnor-Lamotke theorem—has been given by B. J. Sanderson [7] whose techniques are also important for our proceeding.

Proof of theorem. (For the technical details we use the notation explained in §0 of [1].) By an induction over skeletons, it is enough