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KOROVKIN SETS FOR AN OPERATOR ON A SPACE OF CONTINUOUS FUNCTIONS

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We characterize Korovkin sets for sequences of either positive operators or contractive operators converging to an operator T. Properties of both the Korovkin sets and the operator T are given which were previously known only in the case T was the identity operator.

Let $C = C(Q)$ be the Banach space of continuous functions on a first countable, compact Hausdorff space Q. Let $\mathscr{J}$ be a subset of the bounded linear operators $\mathscr{B}(C)$ on C. A subspace X of C is said to be a *$\mathscr{J}$-Korovkin set* for an operator T in $\mathscr{J}$ if for any sequence of operators $\{T_n\}$ in $\mathscr{J}$ the convergence of T_nf to Tf in the uniform norm for all f in X implies the convergence of T_nf to Tf for all f in C. This paper is concerned with $\mathscr{J}$-Korovkin sets when $\mathscr{J}$ consists of either the positive ($f \geqq 0$ implies $Tf \geqq 0$) operators or the contractive ($\|T\| \leqq 1$) operators in $\mathscr{B}(C)$.

The case where T is the identity operator is now classical. See, for instance, Lorentz [3]. In this same paper the extension of the classical theory to the case of arbitrary operators T is mentioned as an open problem. This extension is the subject of the present paper. In case T is the identity operator our results reduce to the classical ones. A number of authors have considered the case where T is a lattice homomorphism between (possibly distinct) vector lattices. The present situation is different since we consider operators with the same domain and range and assume the weaker condition that T either be positive or have norm one. In addition, Cavaretta [1] and Micchelli [4, 5] have considered the case where T is a positive operator, but not necessarily a lattice homomorphism.

Many of the following results have obvious analogues in the case where the operators are assumed to be both positive and contractive.

1. General theory.

Korovkin-type theorems are usually stated for either sequences of positive operators or sequences of contractive operators. Some results about Korovkin sets can be shown in a more general setting. This observation has also been made by Micchelli [5].

For a bounded linear operator T, let T^* denote the adjoint of T. For a point q in Q, let $\hat{q}$ denote the functional in C^* given by evaluation at the point q. Let $\mathscr{L}$ be a subset of C^* and $\mathscr{J}$ be