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SIDON PARTITIONS AND p-SIDON SETS

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Let Γ be a discrete abelian group, and $\Gamma^\wedge = G$ its compact abelian dual group. $E \subset \Gamma$ is called a p-Sidon set, $1 \leqq p < 2$, if $C_E(G)^\wedge \subseteq l^p(E)$. In this paper, a sufficient condition for p-Sidon is displayed and some applications are given.

The notion of p-Sidon sets was first explored in [2]. One of the highlights of [2] was the observation that if E_1 and E_2 are infinite, mutually disjoint sets, and $E_1 \cup E_2$ is dissociate, then $E_1 + E_2$ is 4/3-Sidon, but not $(4/3 - \varepsilon)$-Sidon for any $\varepsilon > 0$. This was subsequently extended in [4]: Let $E_1, E_2, \cdots, E_N$ be infinite and mutually disjoint sets whose union is dissociate. Then $E_1 + E_2 + \cdots + E_N$ is $(2N/N+1)$-Sidon, but not $(2N/(N+1) - \varepsilon)$-Sidon for any $\varepsilon > 0$. The methods in [2] and [4] relied on the theory of tensors, and were based on Littlewood's classical inequality ([6]): Let $(a_{ij})_{i,j=1}^N$ be a complex matrix so that $|\sum a_{ij}s_it_j| \leqq 1$ for any $(s_i)_{i=1}^N$ and $(t_j)_{j=1}^N$ where $|s_i|$, $|t_j| \leqq 1$, $i, j = 1, \cdots, N$. Then, $\sum_i (\sum_j |a_{ij}|^2)^{1/2} \leqq K$, where K is a universal constant (independent of N). In this paper, we do away with the language of tensors, and isolate the ingredients that were essential to the examples of p-Sidon sets constructed thus far (Theorem A in § 1).

In § 2, we give some applications of Theorem A: If $E \subset \Gamma$ is a Sidon set, then $E \times E$ is 4/3-Sidon in $\Gamma \times \Gamma$. We prove also that if $E \subset \Gamma$ is dissociate, then $\underbrace{E \pm E \pm \cdots \pm E}_{N\text{-times}}$ is $(2N/N + 1)$-Sidon. We conclude (§ 3) with some remarks on the connection between harmonic analysis and the metric theory of tensors.

1. Sidon partitions.

DEFINITION 1.1. $\{F_j\}$ is a Sidon partition for $E \subset \Gamma$ if (i) $\cup F_j = E$, and (ii) every bounded function that is constant on F_j can be realized as a restriction to E of a Fourier-Stieltjes transform.

REMARK. A simple category argument shows that there is $C \geqq 1$ so that whenever $\phi \in l^\infty(E)$ is constant on F_j, for all j, and $\|\phi\|_\infty \leqq 1$, then the interpolating measure μ_ϕ in the above definition can be chosen so that $\|\mu_\phi\| \leqq C$.

THEOREM A. *Suppose that $E \subset \Gamma$ can be written as $E =$*