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WEAK HOMOMORPHISMS AND INVARIANTS: AN EXAMPLE

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Notions of weak isomorphism, weak epimorphism, and weak embedding are defined. For countable algebras, these specialize to the ordinary notions. Certain invariants for superatomic Boolean algebras are described. It is shown that the existence or non-existence of weak isomorphisms, weak epimorphisms, and weak embeddings between two such algebras A and B can be decided from the invariants of A and B.

I. Introduction. In [4], Day described certain invariants for superatomic Boolean algebras that refine invariants first introduced by Mazurkiewicz and Sierpinski [6]. Day showed using topological methods that any two countable superatomic Boolean algebras with the same invariants are isomorphic. In [3], Cramer described a partial order $\leqq$ on the Day invariants. He showed, again using topological methods, that the countable algebra A is embeddable in B if and only if the Day invariant of A is $\leqq$ the Day invariant of B, and that the countable algebra B is a homomorphic image of the countable algebra A if and only if the invariant of A is $\geqq$ the invariant of B. Day and Cramer give examples that show the countability assumptions cannot be dropped.

In this paper, we describe notions of weak isomorphism, weak embedding and weak epimorphism that have already been used with success in the study of Abelian torsion groups [2]. We then show that for any two superatomic Boolean algebras A and B, A is weakly isomorphic to B iff A and B have the same Mazurkiewicz-Sierpinski invariant, A is weakly embeddable in B iff the invariant of A is $\leqq$ the invariant of B, and B is a weak image of A iff the invariant of A is $\geqq$ the invariant of B. From these results it is in particular easy to derive the results of Day and Cramer mentioned above.

The motivation for looking at the subject came from infinitary logic, and our first proof of the main result used a certain amount of machinery from that subject. The proof we present here, however, uses only a little elementary algebra. There is a good deal of evidence (see Barwise [1]) that the notion of weak isomorphism is algebraically more natural and better behaved that the notion of isomorphism. Our main result will add a little to that evidence.

II. Weak homomorphisms. Let A, B be algebraic structures