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INSERTION OF A CONTINUOUS FUNCTION

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Necessary and sufficient conditions in terms of lower cut sets are given for the insertion of a continuous function between two comparable real valued functions with a certain pair of a general class of properties. The class of properties is defined by being preserved when added to a continuous function and by being possessed by any constant function.

A property P defined relative to a real valued function on a topological space is a *C property* provided any constant function has property P and provided the sum of a function with property P and any continuous function also has property P. If P_1 and P_2 are C properties, the following terminology is used: (i) A space X has the *weak C insertion property for* (P_1, P_2) if and only if for any functions g and f on X such that $g \leqq f$, g has property P_1 and f has property P_2, then there exists a continuous function h on X such that $g \leqq h \leqq f$. (ii) A space X has the *C insertion property for* (P_1, P_2) if and only if for any functions g and f on X such that $g < f$, g has property P_1 and f has property P_2, then there exists a continuous function h on X such that $g < h < f$. (iii) A space X has the *strong C insertion property for* (P_1, P_2) if and only if for any functions g and f on X such that $g \leqq f$, then there exists a continuous function h on X such that $g \leqq h \leqq f$ and such that if $g(x) < f(x)$ for any x in X, then $g(x) < h(x) < f(x)$. If a space X has the weak C insertion property for (P_1, P_2), necessary and sufficient conditions in terms of lower cut sets are given for the space to have the C insertion property for (P_1, P_2). For a space with the weak C insertion property, a sufficient condition for the space to have the strong C insertion property is given. In certain situations, this condition is necessary. Several insertion theorems, some of which are known, are obtained as corollaries of these two results.

1. Examples of C properties and C insertion.

The following are examples of C properties: lower semicontinuous (lsc), upper semicontinuous (usc), normal lsc, normal usc, continuity, measurable, almost continuous (in the sense of Husain **[5]**), almost continuous (in the sense of Singal and Singal **[12]**), Baire class one. If f_* denotes the lower limit functions of f, then f is normal lsc in case $f = (f^*)_*$ and f is normal usc in case $f = (f_*)^*$; see Dilworth **[3]** for amplification. Observe that any set of functions that satisfies a given C property contains the set of all continuous functions on the space.