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NORMS OF POWERS OF ABSOLUTELY CONVERGENT FOURIER SERIES: AN EXAMPLE

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Let f be defined on T^2 and have an absolutely convergent Fourier series

$$f(e^{i\sigma_1}, e^{i\sigma_2}) = \sum f_m e^{im_1\sigma_1} e^{im_2\sigma_2}.$$

Set $\|f\| = \Sigma |f_m|$ and $\|f\|_2^2 = \Sigma |f_m|^2$. In this paper the asymptotic behavior of $\|f^k\|$, as $k \to \infty$, is studied.

THEOREM 1. *Let f be a continuous function on T^2 such that*

$$(1) \qquad |f(z)| < 1 \ \text{for} \ z \neq (1,1), \quad |z_1| = |z_2| = 1$$

and such that for all σ in some R^2 neighborhood of $(0,0)$

$$(2) \qquad f(e^{i\sigma}) = \exp(i\lambda(\sigma) - \psi(\sigma))$$

where λ is a real-valued linear function defined on R^2 and ψ is a continuous, complex-valued valued function defined on R^2 and satisfying certain smoothness conditions to be defined near the end of §2 *below. Then*

(i) $\|f^k\|_2^2 \leqq ak^{-1/p} \log k$,
(ii) $\sup_m |(f^k)_m| \leqq ak^{-1/p} \log k$,
(iii) $bk^{-1/p} \log k \leqq \sup_m \operatorname{Re}(f^k)_m$,
(iv) $b \log k \leqq \Sigma_m |\operatorname{Re}(f^k)_m|$.

THEOREM 2. *There exists a polynomial f in two complex variables satisfying $f(1,1) = 1$ and*

$$|f(z)| < 1 \ \text{for} \ z \neq (1,1), \quad |z_1| = |z_2| = 1,$$

such that

(i) $\|f^k\| \leqq b \log k$,
(ii) $a \log k \leqq \|f^k\|$,
(iii) $a \log k \leqq \| |f|^k \|$,
(iv) $\|f^k\|_2^2 \leqq b \sup_m |(f^k)_m|$.

In 1970 B. M. Schreiber published smoothness conditions which, for functions defined on T^q, having absolutely convergent Fourier series and