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SOME NONOSCILLATION CRITERIA FOR HIGHER ORDER NONLINEAR DIFFERENTIAL EQUATIONS

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Sufficient conditions for an nth order nonlinear differential equation to be nonoscillatory are given. An essential part of the hypotheses is that a related linear equation be disconjugate.

The linear differential equation

$$x^{(n)} + p(t)x = 0, \tag{1}$$

where $p: [t_0, \infty) \to R$ is continuous, is said to be eventually disconjugate if there exists $T \geqq t_0$ such that no solution of (1) has more than $n-1$ zeros (counting multiplicities) on $[T, \infty)$. A solution $x(t)$ of (1) (or equation (2) below) will be called nonoscillatory if there exists $t_1 \geqq t_0$ such that $x(t) \neq 0$ for $t \geqq t_1$. Equation (1) (or (2)) will be called nonoscillatory if all its solutions are nonoscillatory. Clearly, disconjugacy implies nonoscillation. On the other hand, for $n = 2, 3$ or 4 and either $p(t) > 0$ or $p(t) < 0$, if equation (1) is nonoscillatory, then (1) is eventually disconjugate. Whether this is true for $n > 4$ remains an open question (see Nehari **[11]**).

In this paper we consider the nonlinear differential equation

$$x^{(n)} + q(t)f(t, x, x', \cdots, x^{(n-1)}) = 0 \tag{2}$$

where $q: [t_0, \infty) \to R$ and $f: [t_0, \infty) \times R^n \to R$ are continuous, and obtain some nonoscillation results by making assumptions on the disconjugacy of certain related linear equations. A discussion of disconjugacy criteria for linear differential equations can be found in Coppel **[2]**, Levin **[10]**, Nehari **[11]**, Trench **[12]**, or Willett **[13]**. For a discussion of nonoscillation criteria for second order nonlinear equations we refer the reader to the recent papers of Coffman and Wong **[1]**, Graef and Spikes **[3–5]**, Wong **[14]**, and the references contained therein. There appears to be no known sufficient conditions for nonoscillation of higher order nonlinear equations.

We will assume that there is a continuous function $W: [t_0, \infty) \times R^n \to R$ such that

$$|f(t, u_1, \cdots, u_n)| \leqq W(t, u_1, \cdots, u_n)|u_1| \tag{3}$$

for all $(t, u_1, \cdots, u_n) \in [t_0, \infty) \times R^n$, and