PACIFIC JOURNAL OF MATHEMATICS
Vol. 66, No. 1, 1976

EXTENSIONS OF COMPLETELY REGULAR ORDERED SPACES

T. H. CHOE AND Y. H. HONG

In this paper we introduce the concept of o-completely regular filters on a completely regular ordered space in order to characterize compact ordered spaces and show that for any completely regular ordered space X, Nachbin compactification of X is precisely the strict extension of X with all maximal o-completely regular filters as the filter trace. With this concept, we also define k-compact ordered spaces for every infinite cardinal k, which give rise to a chain of extensive subcategories of the category of completely regular ordered spaces, analogous to the chain given by the category of k-compact spaces, for the different cardinals k.

0. Introduction. Nachbin **[15]** has introduced the concept of completely regular ordered space as a generalization of completely regular topological space and, as a generalization of the Stone-Čech compactification, has shown that the category of compact ordered spaces and continuous isotones is an extensive subcategory (2.2) of the category of completely regular ordered spaces and continuous isotones. Herrlich **[8]** has introduced k-compact spaces for each infinite cardinal and has shown that they form a chain of extensive subcategories of the category of completely regular spaces and continuous maps.

Introducing o-completely regular filters on a completely regular ordered space, it is shown that for a completely regular ordered space X, Nachbin compactification of X is the strict extension of X with all maximal o-completely regular filters as the filter trace. Using this fact, we define k-compact ordered spaces for every infinite cardinal k, and show that categories of k-compact ordered spaces are extensive in the category of completely regular ordered spaces and continuous isotones.

It is known **[11]** that a completely regular space is k-compact iff it is k-closed (3.2) in its Stone-Čech compactification and $\aleph_1$-compact spaces are precisely realcompact spaces, or closed subspaces of powers of the real line R.

In our case, k-compact ordered spaces are precisely those spaces which are k-closed in their Nachbin compactifications, but it is shown that $\aleph_1$-compact ordered space need not be an R-compact ordered space, i.e. a closed subspace of a power of R.

Finally it is noted that by replacing continuous isotones between topological partially ordered spaces with continuous homomorphisms