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THE BOUNDARY BEHAVIOR OF HENKIN'S KERNEL

PATRICK AHERN AND ROBERT SCHNEIDER

In this paper, the boundary behaviour of a reproducing kernel, introduced by Henkin, for strictly pseudoconvex domains is studied. As an application, an improved version of a known result about generators of certain maximal ideals is given.

The boundary behaviour of the Bergmann kernel $B(z, \zeta)$ for a strictly pseudoconvex domain has been studied by Bergmann **[1]** and Hörmander **[5]**. Among other things, they determine the rate at which $B(z, z)$ goes to infinity as z approaches a boundary point of the domain. Another type of reproducing kernel has been introduced by Henkin **[3]** for bounded strictly pseudoconvex domains D, in $\mathbf{C}^n$. Henkin's kernel is of the form $K(\zeta, z)/\Phi^n(\zeta, z)$, where K and Φ are holomorphic in a neighborhood of $\bar{D}$ for each ζ in ∂D, the boundary of D. The denominator Φ has the properties that $\Phi(\zeta, \zeta) = 0$ for all $\zeta \in \partial D$ and that $\Phi(\zeta, z) \neq 0$ if $z \in \bar{D} \setminus \{\zeta\}$. For z near ζ, Φ is given explicitly (up to a nonvanishing factor) in terms of the plurisubharmonic function ρ that defines the domain D. Precise statements about the way $\Phi(\zeta, z)$ approaches zero as z approaches ζ from inside D are given in Henkin's paper **[3]**. We show that this determines the behaviour of the kernel K/Φ^n by showing that $K(\zeta, \zeta) \neq 0$.

It has been proven in **[4]**, **[6]**, **[7]** and **[9]**, that if f is in the space $A(D)$ of functions continuous on $\bar{D}$ and holomorphic in D and if $a \in D$ then there exist functions $g_1, \cdots, g_n \in A(D)$ such that

$$f(z) - f(a) = \sum_{j=1}^{n} (z_j - a_j) g_j(z).$$

This is a solution to a problem originally posed by Gleason **[2]** for the unit ball in $\mathbf{C}^n$. Using Henkin's integral formula and our result on the behaviour of Henkin's kernel we can improve the result just stated in two ways. Firstly, we show that the g_i can be chosen in such a way that the association between f and the n-tuple of functions $(g_1, \cdots, g_n)$ is linear, and secondly we show that the g_i may be also chosen to depend analytically on a as well as on z.

1. Notation. D will always denote a bounded strictly pseudoconvex domain in $\mathbf{C}^n$ defined as $D = \{z : \rho(z) < 0\}$, where ρ is defined and strictly plurisubharmonic in a neighborhood U of $\bar{D}$, such that the gradient of ρ is not zero on the boundary of D. For $\epsilon > 0$ we let