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WIENER INTEGRALS OVER THE SETS BOUNDED BY SECTIONALLY CONTINUOUS BARRIERS

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Let $C_W \equiv C[0, T]$ denote the Wiener space on $[0, T]$. The Wiener integrals of various functionals $F[x]$ over the space C_W are well-known. In this paper we establish formulas for the Wiener integrals of $F[x]$ over the subsets of C_W bounded by sectionally continuous functions.

1. Introduction.

Let $C_W \equiv C[0, T]$ be the Wiener space on $[0, T]$, i.e., the space of all real-valued continuous functions on $[0, T]$ vanishing at the origin. The standard Wiener process $\{X(t) \equiv X(t, \cdot): 0 \leqq t \leqq T\}$ and C_W are related by $X(t, x) = x(t)$ for each x in C_W. Evaluation formulas for the Wiener integral

$$\int_{C_W} F[x] d_W x \equiv E\{F[x]\}$$

of various functionals $F[x]$ are of course well-known (for example see **[7]** for some of these formulas). Now, consider sets of the type

$$\Gamma_f \equiv \left\{ \sup_{0 \leqq t \leqq T} X(t) - f(t) < 0 \right\}$$
$$= \left\{ x \in C_W: \sup_{0 \leqq t \leqq T} x(t) - f(t) < 0 \right\}$$

where $f(t)$ is sectionally continuous on $[0, T]$ and $f(0) \geqq 0$.

It is well-known that for $b \geqq 0$

$$P[\Gamma_b] = 2\Phi(bT^{-1/2}) - 1$$

and

$$P[\Gamma_{at+b}] = \Phi[(aT + b)T^{-1/2}] - e^{-2ab}\Phi[(aT - b)T^{-1/2}]$$

where Φ is the standard normal distribution function. In **[3]**, **[5]**, and **[6]** more general functions $f(t)$ are considered and formulas given for the probabilities of the sets Γ_f.

The main purpose of this paper is to derive formulas for Wiener integrals over the sets Γ_f. In §2 we state and prove the main results, while in §3 we discuss some applications and examples.