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AN IMBEDDING THEOREM FOR INDETERMINATE HERMITIAN MOMENT SEQUENCES

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Hermitian moment sequences are generalizations of classical power moment sequences to bounded operators on a Hilbert space. The main result is that every indeterminate Hermitian moment sequence on a complex Hilbert space can be imbedded in a determinate Hermitian moment sequence on an enlarged Hilbert space in the sense that the first sequence is a compression of the second. This implies the existence of determinate Hermitian moment sequences which, when compressed, are indeterminate and leads to the following questions: Which orthogonal projections on the Hilbert space give rise to determinate compressions of a fixed, determinate sequence? What structure do these projections induce on the underlying Hilbert space?

Background. Let $\mathscr{H}$ be a complex Hilbert space, with inner product $(\cdot,\cdot)$ and norm $\|\cdot\|$, and let $B(\mathscr{H})$ be the set of all bounded linear operators on $\mathscr{H}$. A sequence $\{T_j\}_{j=0}^{\infty}$, $T_j \in B(\mathscr{H})$, is said to be an *Hermitian moment sequence* (cf. J. S. MacNerney **[4]**) if there exists a positive operator-valued measure (cf. Berberian **[2]**), $\mu(\cdot)$, defined on the Borel sets of $(-\infty,\infty)$, such that

$$T_j = \int_{-\infty}^{\infty} t^j d\mu(t), \qquad j = 0, 1, 2, \cdots. \tag{1}$$

Necessary and sufficient conditions for a sequence of operators to be of the form (1) and, in addition, be such that $\mu(\cdot)$ has support over a finite interval were first given by B. Sz.-Nagy **[6]**. General necessary and sufficient conditions for a sequence to be of the form (1), with no restrictions on the support of $\mu(\cdot)$, were first given by J. S. MacNerney **[4]**.

An Hermitian moment sequence (1) will be said to be *determinate* if $\mu(\cdot)$ is unique, and *indeterminate* otherwise (cf. Akhiezer **[1]**, Dubois–Violette **[3]**, and Shohat and Tamarkin **[7]**).

Main result. A determinate moment sequence $\{T_j\}$ can be "imbedded" in an indeterminate moment sequence on $\mathscr{K} = \mathscr{H} \oplus \mathscr{H}$. For example, let $\{a_j\}_{j=0}^{\infty}$ be a scalar-valued indeterminate moment sequence, I be the identity operator on $\mathscr{H}$, and P be the orthogonal projection from $\mathscr{K}$ to $0 \oplus \mathscr{H} = \mathscr{H}$. The sequence,