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ON A CLASS OF UNBOUNDED OPERATOR ALGEBRAS II

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In this paper we continue our study of unbounded operator algebras. On the basis of the space $L^{\omega}[0,1]$ introduced by R. Arens [1] we define and investigate unbounded Hilbert algebras. The primary purpose of this paper is to investigate the relation between unbounded Hilbert algebras and $EW^{\#}$-algebras and the structure of some $EW^{\#}$-algebras.

1. Introduction. In a previous paper **[10]** we began our study of $EW^{\#}$-algebras. For the definitions and the basic properties concerning $EW^{\#}$-algebras is referred to **[10]**. It is well known that semifinite von Neumann algebras are related to Hilbert algebras. That is, if $\mathscr{D}_0$ is a Hilbert algebra, then the left von Neumann algebra $\mathscr{U}_0(\mathscr{D}_0)$ is defined and $\mathscr{U}_0(\mathscr{D}_0)$ is a semifinite von Neumann algebra and conversely if $\mathfrak{A}$ is a semifinite von Neumann algebra, then there exists a Hilbert algebra $\mathscr{D}_0$ such that $\mathfrak{A}$ is isomorphic to the left von Neumann algebra $\mathscr{U}_0(\mathscr{D}_0)$. In this paper we study the above facts about $EW^{\#}$-algebras. So, our starting point will be the extension of Hilbert algebras.

DEFINITION 1.1. Let $\mathscr{D}$ be a pre-Hilbert space with inner product $(\ |\ )$ and a $*$-algebra. If $\mathscr{D}$ satisfies the following conditions (1) $\sim$ (3);

(1) $(\xi \mid \eta) = (\eta^* \mid \xi^*)$, $\quad \xi, \eta \in \mathscr{D}$;

(2) $(\xi\eta \mid \zeta) = (\eta \mid \xi^*\zeta)$, $\quad \xi, \eta, \zeta \in \mathscr{D}$;

By (2) we define $\pi(\xi)$ and $\pi'(\eta)$ by;

$$\pi(\xi)\eta = \pi'(\eta)\xi = \xi\eta, \quad \xi, \eta \in \mathscr{D}.$$

Then $\pi(\xi)$ and $\pi'(\eta)$ are closable operators on $\mathscr{D}$ and we have $\pi(\xi)^* \supset \pi(\xi^*)$ and $\pi'(\eta)^* \supset \pi'(\eta^*)$. We call π (resp. π') the left (resp. right) regular representation of $\mathscr{D}$.

(3) Putting

$$\mathscr{D}_0 = \{\xi \in \mathscr{D};\ \pi(\xi) \text{ is continuous}\},$$

$\mathscr{D}_0^2$ is dense in $\mathscr{D}$, then $\mathscr{D}$ is called an unbounded Hilbert algebra over $\mathscr{D}_0$. In particular, if $\mathscr{D}_0 \neq \mathscr{D}$, then $\mathscr{D}$ is called a pure unbounded Hilbert algebra over $\mathscr{D}_0$.

In §2 we investigate the properties of unbounded Hilbert algebras and we introduce examples of such unbounded Hilbert algebras ($L^{\omega}[0,1]$,