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AUTOMORPHISM GROUPS OF UNIPOTENT GROUPS OF CHEVALLEY TYPE

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Let G be a quasi-simple algebraic group defined and split over the field k. Let V be a maximal k-split unipotent subgroup of G and $\mathrm{Aut}(V)$ the group of k-automorphism of V. The structure of $\mathrm{Aut}(V)$ is determined and the obstructions to making $\mathrm{Aut}(V)$ algebraic when $\mathrm{char}\, k > 3$ are made explicit. If G is not of type A_2, then $\mathrm{Aut}(V)$ is solvable.

Introduction. In [5] Hochschild and Mostow showed that the automorphism group of a unipotent algebraic group defined over a field k of characteristic zero carries the structure of an algebraic k-group. For example if V is a vector group over $\mathbf{C}$, then $\mathrm{Aut}_{\mathbf{C}}(V) = \mathrm{GL}(n, \mathbf{C})$. For more complicated unipotent groups—even over $\mathbf{C}$—little seems to be known about the actual structure of the automorphism group. On the other hand, it was shown by Sullivan in [8] and again by this author in [3] that the Hochschild–Mostow result never holds in positive characteristics when the dimension of the given unipotent group is greater than one.

In [4] Gibbs determined generators for the (abstract) automorphism group of $V(k)$—the k-rational points of a maximal k-split unipotent subgroup V of any k-split simple algebraic group. The characteristic of the field k was assumed distinct from 2 or 3, but no other assumptions on the field k were made. We refer to such groups V as *unipotent groups* of *Chevalley type*. The purpose of this paper is two-fold:

1. To determine the automorphism groups in characteristic zero of unipotent groups of Chevalley type; and
2. To exhibit the obstructions to making these groups algebraic in positive characteristics.

Let $Aut_V(k)$ denote the group of k-automorphisms of the unipotent k-group of Chevalley type V. We show (2.9) that there is an exact sequence

$$1 \to N(k) \to Aut_V(k) \to H(k) \to 1$$

such that

(i) $H(k)$ is the group of k-rational points of an algebraic k-group H.

(ii) $N(k) = 0$ if $\mathrm{char}\, k = 0$, and $N(k) = \coprod_{n=1}^{\infty} G_a(k)$ if $\mathrm{char}\, k > 3$.

(iii) The above sequence splits and $Aut_V(k)$ is the semi-direct product of $N(k)$ and $H(k)$.