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INCLUSION RELATIONS BETWEEN POWER METHODS OF LIMITATION

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Let $p(x) = \sum p_k x^k$ be a power series with $p_k(k = 0, 1, \cdots)$ complex numbers and $0 < \rho_p \leqq \infty$ its radius of convergence, and assume that $P(x) \neq 0$ for $0 \leqq \alpha_p \leqq x < \rho_p$. The power method of limitation, P, is defined by

$$\lim_p s = \lim_{x \to \rho_p -} \sum_{k=0}^{\infty} p_k s_k x^k / P(x) \quad (x \text{ real})$$

(provided the series converges in $[\alpha_p, \rho_p)$ and the limit exists and is finite). Abel and Borel methods are the best known power methods. In this article inclusion relations between two power methods are investigated. Several theorems are proved, which lead to necessary and sufficient conditions, for inclusion, that are correct under some fairly moderate restrictions.

1. Introduction. Let $P(x) = \sum p_k x^k$ be a power series with $p_k(k = 0, 1, \cdots)$ complex numbers and $0 < \rho_p \leqq \infty$ its radius of convergence, and assume that $P(x) \neq 0$ for $0 \leqq \alpha_p \leqq x < \rho_p$. The *power method of limitation*, P (see Włodarski [19] and Birkholc [2]), is defined by

$$\lim_p s = \lim_{x \to \rho_p -} \sum_{k=0}^{\infty} p_k s_k x^k / P(x) \qquad (x \text{ real})$$

(provided the series convergences in $[\alpha_p, \rho_p)$ and the limit exists and is finite).

The power method Q is defined analogously by $Q(x) = \sum q_k x^k$ and parameters α_q, ρ_q.

The best known power methods are the Abel method and the Borel exponential method. Other power methods which appear in the literature are A_λ, L and (B, a, b) (for more details see next section).

We are concerned here with inclusion relations of the form $P \subseteqq Q$. There are several results in the literature in this direction. Thus, Borwein proved (see [4], [5] and [8]) that $A_\lambda \subseteqq A_\mu$, $A_\mu \nsubseteqq A_\lambda$ provided $-1 < \mu < \lambda$, that $A_\lambda \subseteqq L$, $L \nsubseteqq A_\lambda$ provided $\lambda > -1$ and that $(B, a, \beta) \subseteqq (B, a, b)$ provided $a > 0$, $-\infty < \beta \leqq b < +\infty$.

Other results, obtained by Borwein [4], [8] and Hoischen [12], are of a more general nature. Both authors investigated inclusion relations between power methods whose coefficients, $\{p_k\}$, $\{q_k\}$, are assumed, a priori, to be related by some particular cases of the relation