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v-PREHOMOMORPHISMS ON INVERSE SEMIGROUPS

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A mapping θ of an inverse semigroup S into an inverse semigroup T is called a v-prehomomorphism if, for each $a, b \in S$, $(ab)\theta \leqq a\theta b\theta$ and $(a^{-1})\theta = (a\theta)^{-1}$. The congruences on an E-unitary inverse semigroup $P(G, \mathscr{X}, \mathscr{Y})$ are determined by the normal partition of the idempotents, which they induce, and by v-prehomorphisms of S into the inverse semigroup of cosets of G.

Inverse semigroups, with v-prehomomorphisms as morphisms, constitute a category containing the category of inverse semigroups, and homomorphisms, as a coreflective subcategory. The coreflective map $\eta: S \rightarrow V(S)$ is an isomorphism if the idempotents of S form a chain and the converse holds if S is E-unitary or a semilattice of groups. Explicit constructions are given for all v-prehomomorphisms on S in case S is either a semilattice of groups or is bisimple.

0. Introduction. A mapping θ of an inverse semigroup S into an inverse semigroup T is called a v-prehomomorphism if, for each $a, b \in S$, $(ab)\theta \leqq a\theta b\theta$ and $(a^{-1})\theta = (a\theta)^{-1}$. Thus, if S and T are semilattices, a v-prehomomorphism is just an isotone mapping of S into T. N. R. Reilly and the present author have shown that the E-unitary covers of an inverse semigroup S are determined by v-prehomomorphisms with domain S. In the first section of this paper, we show that the congruences on an E-unitary inverse semigroup $S = P(G, \mathscr{X}, \mathscr{Y})$ are determined by the normal partition of the idempotents, which they induce, and by v-prehomomorphisms of S into the inverse semigroup of cosets of G. The remainder of the paper is concerned with the problem of constructing v-prehomomorphisms on an inverse semigroup S.

In §2, it is shown that inverse semigroups and v-prehomomorphisms constitute a category which contains the category of inverse semigroups and homomorphisms as a coreflective subcategory. Thus, for each inverse semigroup S, there is an inverse semigroup $V(S)$ and a v-prehomomorphism $\eta: S \rightarrow V(S)$ with the property that every v-prehomomorphism with domain S is the composite of η with a homomorphism with domain $V(S)$. It is shown that η is an isomorphism if the idempotents of S form a chain and that the converse holds if S is E-unitary or a semilattice of groups.

Section 3 is concerned with the situation when S is a simple inverse semigroup. It is shown that, in this case, $V(S)$ is also simple, but it need not be bisimple even if S is bisimple. Indeed, if S is