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ORLICZ SPACE CONVERGENCE OF MARTINGALES OF RADON-NIKODYM DERIVATIVES GIVEN A σ-LATTICE

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Let $\{M_k\}$ be an increasing sequence of sub σ-lattices of a σ-algebra $\mathscr{A}$, and let M be the σ-lattice generated by $\bigcup_k M_k$. Let $L\Phi$ be an associated Orlicz space of $\mathscr{A}$-measurable functions, where Φ does not necessarily satisfy the Δ_2-condition. Given $h \in L\Phi$, let f_k be the Radon-Nikodym derivative of h given M_k. Necessary and sufficient conditions are given on h to insure that $\{f_k\}$ converges in $L\Phi$ to f, where f is the Radon-Nikodym derivative of h given M. The situation where f is valued in a Banach space with basis is also examined.

1. Introduction. If λ and μ are countably additive set functions defined on a σ-lattice of sets, then the Radon-Nikodym derivative of λ with respect to μ has been defined by Johansen [4]. We may consider this derivative as a conditional expectation of a function with respect to the σ-lattice in the case where λ is absolutely continuous with respect to μ. Hence we may define martingales in this setting. The relation between martingales and Orlicz spaces has been studied by Darst and DeBoth [3] in the case where the Orlicz function Φ satisfied the Δ_2-condition. In this paper we drop the Δ_2-condition and give necessary and sufficient conditions for all martingales to converge to the appropriate function. We also consider the extension of this theory to Banach space valued set functions.

2. Notation. Let M be a sub σ-lattice of a σ-algebra $\mathscr{A}$ of subsets of a nonempty set Ω, and let λ and μ be countably additive, real valued set functions defined on $\mathscr{A}$. Then f is a derivative of λ with respect to μ on M if f is an extended real-valued function defined on Ω such that

(1) f is M-measurable ($[f > a]$ belongs to M for every real a)

(2) $\lambda(A \cap [f < b]) \leqq b\mu(A \cap [f < b])$ for all $A \in M$, $b \in R$.

(3) $\lambda(B^c \cap [f > a]) \geqq a\mu(B^c \cap [f > a])$ for all $B \in M$, $a \in R$.

Now suppose μ is a finite, nonnegative measure on $\mathscr{A}$, and $h \in L^1(\Omega, \mathscr{A}, \mu)$. Let $\lambda(E) = \int_E h d\mu$ for $E \in \mathscr{A}$. Then λ is a bounded signed measure on $\mathscr{A}$. If f is the Radon-Nikodym derivative of λ with respect to μ on M, then we use the notation $f = E(h, M)$. This notation is used since f is the conditional expectation of h given M