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A UNIQUENESS THEOREM FOR TEMPERED INVARIANT EIGENDISTRIBUTIONS

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Let G be a real reductive Lie group and π a tempered invariant eigendistribution on G. Given a natural ordering on the set of conjugacy classes of Cartan subgroups of G, π is called extremal if it has a unique maximal element in its support. T. Hirai has proved for a restricted class of real simple Lie groups that if π is extremal and satisfies certain regularity conditions, it is uniquely determined by its restriction to the maximal element in its support. The purpose of this paper is to show that Hirai's theorem is true without restriction of the type of G.

1. Introduction. Let G be a connected, acceptable, real reductive Lie group with compact center. Let π be a tempered invariant eigendistribution on G. Then π can be realized as a locally summable function, π', analytic on the dense open set G' of regular elements of G. [1] This function is uniquely determined by its restrictions to a complete set of representatives of Car(G), the set of conjugacy classes of Cartan subgroups of G.

The function π' can be quite complicated on the various Cartan subgroups. However, there is a natural ordering on Car(G) such that, if $[H] \in \text{Car}(G)$ is a maximal element for which $\pi'|_H \neq 0$, then there exist functions ε_R^H and Δ^H on H such that $k^H = \varepsilon_R^H \Delta^H \pi'|_H$ is analytic on all of H and is given by a simple Weyl character type formula [5, vol. II, p. 60-62]. Thus it is useful to know when π is uniquely determined by the restrictions of π' to maximal Cartan subgroups in its support.

For example, if π is the character of a discrete series representation of G, then G has a compact Cartan subgroup B which belongs to the unique maximal conjugacy class in Car(G). Harish-Chandra proved in [2] that π is the unique tempered invariant eigendistribution with given eigenvalue and given formula on B.

In two recent papers [3, 4], Hirai studies the space of tempered invariant eigendistributions on G, and proves a theorem which is a natural generalization of the theorem of Harish-Chandra. However, there is a crucial lemma for this theorem which is only stated for G a simple real Lie group of classical type or of type G_2. Hirai doesn't give a proof of this lemma, but claims it is proved by long but elementary case by case arguments.

The purpose of this paper is to give a general proof of Hirai's