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THE COHOMOLOGICAL DIMENSION OF A n-MANIFOLD IS $n+1$

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It is known that any one-dimensional topological manifold is of cohomological Dimension two. The present paper is devoted to the proof of the conjecture that the cohomological Dimension of any topological n-manifold is $n+1$.

Introduction. Let ϕ be a family of supports on a topological space X. The largest integer n (or ∞) for which there exists a sheaf $\mathscr{A}$ of abelian groups on X such that the Grothendieck cohomology groups $H^n_\phi(X, \mathscr{A}) \neq 0$ is called the *cohomological ϕ-dimension* ($\dim_\phi X$) of X. The supremum of all ϕ-dimensions when ϕ runs over all the families of supports on X is called the *cohomological Dimension* ($\operatorname{Dim} X$) of X. The *extent* $E(\phi)$ of a family of supports ϕ is defined to be the union of all members of ϕ. It is then known that if ϕ and ψ are two paracompactifying families of supports on X such that $E(\phi) \subset E(\psi)$ then $\dim_\phi(X) \leqq \dim_\psi(X)$. It follows, therefore, that if ϕ varies over all those paracompactifying families of supports on X whose extents are equal to X then $\dim_\phi(X)$ is independent of ϕ and is called the *cohomological dimension* ($\dim X$) of X. Thus if a space X admits a paracompactifying family of supports with extent equal to X then $\dim X$ makes sense. Let us call a topological space to be *locally paracompact Hausdorff* if each point of the space has a closed paracompact Hausdorff neighbourhood in it. Then one can easily see that for any space X, $\dim X$ makes sense if and only if X is locally paracompact Hausdorff. An intereting relation between $\dim X$ and $\operatorname{Dim} X$ when X is a nice space is given by the following: *If X is locally completely paracompact Hausdorff then* $\dim X = \operatorname{Dim} X$ or $\dim X = \operatorname{Dim} X - 1$. An open problem as to which one it is, was solved in the case of one-dimensional topological manifolds in [2]. The main objective of this paper is to prove the conjecture of [2] by showing that if X is a topological n-manifold then $\operatorname{Dim} X = n+1$ for any $n \geqq 1$.

1. Preliminaries. By a sheaf in this paper we shall mean a sheaf of abelian groups. If $\mathscr{A}$ is a sheaf on a space X, $\mathscr{C}^0(X, \mathscr{A})$ will denote the sheaf on X generated by the presheaf $U \rightarrow \mathrm{C}^0(U, \mathscr{A})$ where $\mathrm{C}^0(U, \mathscr{A})$ is abelian group of all sections (not necessarilly continuous) of $\mathscr{A}$ on the open set U of X. Similarly, $\mathscr{F}^1(X, \mathscr{A}) = \mathscr{F}^1$ will denote the quotient sheaf $\mathscr{C}^0(X, \mathscr{A})/\mathscr{A}$ on X. For each positive integer n we define inductively