

TOTAL POSITIVITY OF CERTAIN REPRODUCING KERNELS

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In this paper we study the total positivity of various kernels, especially reproducing kernels of Hilbert spaces of analytic functions. We do so by employing a familiar device known as the "composition formula of Pólya and Szegő." Using this formula we are able to give a short proof of the variation diminishing property of a generalized analogue of the la Vallée Poussin means. This generalizes earlier work of Pólya and Schoenberg and recent work of Horton. Our method is also based on the isometrical image of the reproducing kernel called the generating function. The reproducing kernel is then expressed as a composition of two generating functions so that the problem is reduced to investigating the total positivity of the generating function. This method extends earlier work and yields many new reproducing kernels which are total positive.

1. Introduction. The theory of total positivity and more generally the theory of sign regularity have been extensively applied in various fields of mathematics and in particular in the theory of approximation [9, 10]. In a previous paper [5] it was shown that the optimality of a quadrature formula is closely connected with the notion of the total positivity of the reducing kernel of the functions determining the formula (cf. Karlin [10]). In [5] the notion of total positivity was extended in a natural manner to domains in the complex plane. For simply connected domains, for which the reproducing kernel is an automorphic form of arbitrary weight, it was shown that the reproducing kernel is indeed totally positive thereby yielding a differential geometric interpretation of total positivity. It was also shown that in general, reproducing kernels of multiply connected domains are not totally positive. The methods in [5] however cannot be applied to reproducing kernels which are not automorphic forms and the purpose of this paper is to establish the total positivity of such kernels. We do this by employing a familiar device (Karlin [9, p. 98]) known as the "composition formula of Pólya and Szegő." As mentioned by Karlin, this is the only device known to us as a binary operation, that permits us to construct a totally positive kernel from two such kernels.

Using the above composition formula and an explicit formula for Jacobi polynomials due to Bateman [3], we will give a very short proof of a theorem, proved first by Horton [8], on the variation