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HOMOTOPY THEORETIC CONSEQUENCES OF N. LEVITT'S OBSTRUCTION THEORY TO TRANSVERSALITY FOR SPHERICAL FIBRATIONS

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The main goal of this paper is a detailed analysis of the problem of imposing a topological bundle structure on a spherical fibre space over a simply connected base. The method involves a careful study of the notion of fibre homotopy transversality due to N. Levitt. The point is, a topological disc bundle satisfies strong transversality properties for maps from manifolds to the associated Thom space. These properties can be formulated at least for spherical fibre spaces. Thus, obstructions to transversality can be interpreted as obstructions to imposing a topological bundle structure on a spherical fibre space. It turns out that over a simply connected base the obstructions to transversality coincide exactly with the obstructions to a topological structure.

The obstructions to transversality for a spherical fibre space ξ can be interpreted as obstructions to a deformation of the identity map on the Thom space $T\xi$ to a certain subcomplexes $W\xi$. The fibre of the map $W\xi \to T\xi$ isa sp ace with a suitable iterated loop space homotopy equivalent to G/TOP. The total obstruction to transversality becomes the obstruction to a $KO \otimes \boldsymbol{Z}[1/2]$ orientation of the Thom space $T\xi$, mixed with certain cohomology classes of $T\xi$, $\widetilde{\mathscr{L}} \in H^{4*+1}(T\xi, \boldsymbol{Z}_{(2)})$ and $\widetilde{\mathscr{K}} \in H^{4*-1}(T\xi, \boldsymbol{Z}/2)$. These obstructions are then also interpretable as the obstructions to lifting in the fibration sequence $G/TOP \to BSTOP \to BSG$.

In this introduction, we give a rather detailed outline of our results and describe the relationship with work of others, particularly, N. Levitt [10], F. Quinn [18], and L. Jones [8]. Suppose that $\pi: \xi \longrightarrow B\xi$ is a spherical fibre space, with Thom space $T\xi$, and let $f: M \longrightarrow T\xi$ be a map from a PL manifold to $T\xi$. The primary question is, very roughly, when can one deform f, so that $f^{-1}(B\xi) \subset M$ is a Poincaré duality space?

If $\xi \longrightarrow B\xi$ is a (block) PL sphere bundle, the answer is always. In fact, in this case we may deform f so that $L = f^{-1}(B\xi) \subset M$ is a PL submanifold of M, with a tubular neighborhood $V = f^{-1}(D\xi)$, where $D\xi$ is the associated PL block bundle of ξ. V is a block bundle over L and f induces a bundle map