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# INTEGRALS OF FOLIATIONS ON MANIFOLDS WITH A GENERALIZED SYMPLECTIC STRUCTURE

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**Let $M$ be a $C^\infty$ manifold of dimension $m$ and $E$ an integrable subbundle (foliation) of the tangent bundle $TM$. We are interested in structures on the set of all local integrals of $E$. For example, if $M$ is a symplectic manifold then the Poisson brackets operation on the set $C^\infty_{\text{loc}}$ of all local functions of $M$ defines an algebraic structure on $C^\infty_{\text{loc}}$. Earlier authors have called such structures "function groups." In particular, if $X_H$ is a nonvanishing Hamiltonian vector field, then $X_H$ defines a foliation $E$ of $M$ and the set of all local integrals of $E$ is also a function group.**

The Poisson brackets operation can be defined on manifolds with somewhat less restrictive requirements than that of being symplectic. Other authors such as S. Lie and C. Carathéodory [4] have studied this more general notion of Poisson brackets in the classical local setting. Hermann [9, p. 31] has indicated how to extend the definition of Poisson brackets to functions on manifolds having a closed 2-form $\omega$ of constant rank (Recall that $M$ is called symplectic if $\omega_p$ has rank $m$ for each $p \in M$).

The paper is largely self-contained, but does require the use of the following basic identities:

$$L_X Y = [X, Y] , \quad L_X = i_X d + d i_X , \quad L_X i_Y - i_Y L_X = i_{[X,Y]} .$$

The proofs of these identities may be found in Chapter IV of the first volume of [7]. Other undefined terms appear either in [1] or [7].

**1. Generalized symplectic structures on manifolds.** Let $M$ be a $C^\infty$ manifold of dimension $m$ and let $\omega$ be a closed 2-form on $M$. Recall that the kernel of a 2-form $\omega$ can be defined at each point $p \in M$ by

$$\begin{aligned}\ker \omega_p &= \{v \in M_p \mid \omega(v, M_p) = 0\} \\ &= \{v \in M_p \mid \omega(M_p, v) = 0\} .\end{aligned}$$

The rank of $\omega$ at $p$ is defined to be the rank of the bilinear map $\omega_p: M_p \times M_p \to R$. Of course, since $\omega_p$ is a skew-symmetric bilinear map its rank is the even integer $m - \dim(\ker \omega_p)$.

Let $\Gamma$ denote the set of sections of $TM$ and $\Gamma^*$ the set of sections of $T^*M$. Define $\alpha: \Gamma \to \Gamma^*$ by