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REGULAR LATTICE MEASURES: MAPPINGS AND SPACES

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We prove in this paper a very general measure extension theorem which has as corollaries many recent, significant extension theorems in the literature. We apply these results to the question of when there is a well behaved map from the σ-smooth lattice regular measures on one set to the σ-smooth lattice regular measures on a second set. After developing these general theorems we specialize consideration to two valued latitce regular measures and obtain in a new and consistent manner many important mapping and subspace theorems on the preservation of different types of repleteness including results of Dykes, Hager, Isiwata, Moran, Varadarajan, Gillman, Jerrison and others.

Introduction. In earlier papers [6], [7], [49], we have developed for an abstract set X and a given lattice $\mathscr{L}$ of subsets, the concept of $\mathscr{L}$-repleteness. This concept, as well as others such as $\mathscr{L}$-compact, $\mathscr{L}$-countably compact, etc., considered by Alexandroff [1-3], Meyer [41], Marczewski [39], Topsøe [50], Frolik [19], and others, we have expressed measure theoretically in terms of two valued $\mathscr{L}$-regular measures. This is advantageous; for besides being analytically simpler to work with than with filters, many theorems in this form can be generalized naturally to arbitrary $\mathscr{L}$-regular measures, and the topological settings extended from Wallman topologies to vague topologies.

The notion of $\mathscr{L}$-replete includes as special cases: real compact, Borel complete [24], α-complete [15], etc., and in [6], we developed measure-theoretic results to show systematically how to obtain repleteness interrelations. Here, we are concerned with mapping and subspace problems. The mapping questions are of the type: Given $T: X \to Y$ which is well-behaved (see §5 for details) with respect to two lattices $\mathscr{L}_1$, $\mathscr{L}_2$ of subsets of X and Y respectively, when does T induce a well-behaved mapping $T^{**}: MR(\sigma, \mathscr{L}_1) \to MR(\sigma, \mathscr{L}_2)$, where, in general, $MR(\sigma, \mathscr{L})$ designates the σ-smooth $\mathscr{L}$-regular measures on a set X with respect to a lattice, $\mathscr{L}$, of subsets? We develop the major results of this type in §5, and when applied to the special case of two-valued measures, we get as corollaries important subspace and mapping results of Frolik [21], Ishiwata [31], Dykes [15, 16], Moran [42], Hager [24], Mrowka [43], Varadarajan [51], Gordon [23], as well as classical results in realcompactness as