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ON THE HAUSDORFF–YOUNG THEOREM FOR INTEGRAL OPERATORS

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A sharp inequality of Hausdorff–Young type is proved for integral operators. Applications are made in operator theory and in harmonic analysis on locally compact groups.

In recent work the author stated an inequality of Hausdorff–Young type for integral operators which proved to be useful in obtaining L^p estimates on certain locally compact unimodular groups. The present paper is devoted to a closer analysis of that inequality together with some applications to operator theory and to L^p-Fourier analysis on locally compact groups.

In the first place, the proof given previously in **[15** I], for the inequality is incomplete, so this paper will begin in §2 with a correct proof of the inequality (Theorem 1). Also shown in §2 is the nonexistence of extremal functions in a particular instance (Prop. 8). In §3 the results of §2 are applied to obtain estimates for the norm of the L^p-Fourier transform on certain unimodular groups. Here some of the machinery from **[10]** is used in the examples, one class of which (Prop. 13) does not depend on Theorem 1. This has happened before, see **[15** I: §3]. For certain members of this class however, it is shown that a better estimate can be obtained using Theorem 1 (Prop. 15). In §4 the study of Hausdorff–Young inequalities on nonunimodular groups is initiated. In view of the recent work on Plancherel formulas for nonunimodular groups such inequalities with constant 1 might be considered routine. It is shown here using Theorem 1, that the natural Hausdorff–Young inequality on the "$ax + b$" group has a constant less than 1 (Prop. 19). In §5 an operator valued analog of the Fourier transform on Abelian groups is introduced, which is motivated by preceding sections, and it is shown, using Theorem 1, that it behaves in some respects like the Fourier transform (Prop. 20).

2. The Hausdorff–Young Theorem for integral operators. Let X be a σ-finite measure space, $k \in L^2(X \times X)$, and let K be the integral operator with kernel k, i.e.,

$$(1) \qquad Kf(x) = \int k(x, y) f(y) dy, \qquad f \in L^2(X), \qquad x \in X.$$