

GENERALISED QUASI-NÖRLUND SUMMABILITY

PREMALATA MOHAPATRO

Just as (N, p, q) generalises Nörlund methods, so also, in this paper we define generalised quasi-Nörlund Method (N^*, p, q) generalising the quasi-Nörlund method due to Thorpe.

To begin with, we have determined the inverse of a generalised quasi-Nörlund matrix in a limited case. Besides, limitation Theorems for both ordinary and absolute (N^*, p, q) summability have been established.

Finally we have established an Abelian Theorem (the main theorem) for $(N^*, p, q) \Rightarrow (J, q)$, where (J, q) is a power series method which reduces to the Abel method (A) for $q_n = 1$ (all n).

1. Vermes [10] pointed out that there is a close relation between the summability properties of a matrix $A = (a_{nk})$ regarded as a sequence to sequence transformation and those of its transpose $A^* = (a_{kn})$ regarded as a series to series transformation.

Suppose that A is a sequence to sequence transformation and further that

$$\sum_{k=0}^{\infty} a_{nk} = 1 \quad \text{for all } n,$$

then by using Theorems of regularity (see Hardy [5], Theorem 2) and absolute regularity (see Knopp and Lorentz [6]) we see that A^* is an absolutely regular series to series transformation.

Conversely, given any absolutely regular series to series method $C = (c_{nk})$, its transpose C^* is regular as a sequence to sequence method provided that

$$c_{nk} \rightarrow 0 \quad \text{as } k \rightarrow \infty \quad \text{for fixed } n.$$

We can also see that if A is absolutely regular and the above condition is satisfied then A^* is regular and the converse also holds.

We shall call A^* the quasi-method associated with A and remember that, it is a series to series transformation.

Kuttner [7] defined quasi-Cesàro summability and investigated its main properties as a quasi-Hausdorff transformation (see also Ramunujan [8] and White [11]). Thorpe [9] defined quasi-Nörlund (quasi-Riesz) summability.

Just as (N, p, q) generalises Nörlund methods, so also we can define generalised quasi-Nörlund method (N^*, p, q) generalising the quasi-Nörlund methods. We give the definition in the following manner: