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E-UNITARY COVERS FOR INVERSE SEMIGROUPS

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An inverse semigroup is called E-unitary if the equations $ea = e = e^2$ together imply $a^2 = a$. In a previous paper, the first author showed that every inverse semigroup has an E-unitary cover. That is, if S is an inverse semigroup, there is an E-unitary inverse semigroup P and an idempotent separating homomorphism of P onto S. The purpose of this paper is to consider the problem of constructing E-unitary covers for S.

Let S be an inverse semigroup and let F be an inverse semigroup, with group of units G, containing S as an inverse subsemigroup and suppose that, for each $s \in S$, there exists $g \in G$ such that $s \leqq g$. Then $\{(s, g) \in S \times G: s \leqq g\}$ is an E-unitary cover of S. The main result of §1 shows that every E-unitary cover of S can be obtained in this way. It follows from this that the problem of finding E-unitary covers for S can be reduced to an embedding problem. A further corollary to this result is the fact that, if P is an E-unitary cover of S and P has maximal group homomorphic image G, then P is a subdirect product of S and G and so can be described in terms of S and G alone. The remainder of this paper is concerned with giving such a description.

1. *E*-unitary covers. An inverse semigroup is called E-unitary if the equations $ea = e = e^2$ together imply $a^2 = a$. It was shown in **[4]** that every inverse semigroup S has an E-unitary cover in the sense that there is an E-unitary inverse semigroup P together with an idempotent separating homomorphism θ of P onto S. It was further shown in **[5]** that every E-unitary inverse semigroup is isomorphic to a $P(G, \mathscr{X}, \mathscr{Y})$ where $\mathscr{X}$ is a down directed partially ordered set with $\mathscr{Y}$ an ideal and subsemilattice of $\mathscr{X}$ and where G acts on $\mathscr{X}$ by order automorphisms in such a way that $\mathscr{X} = G\mathscr{Y}$; see **[5]** for details. The group G in $P = P(G, \mathscr{X}, \mathscr{Y})$ is isomorphic to the maximum group homomorphic image P/σ of P where

$$\sigma = \{(a, b) \in P \times P: ea = eb \quad \text{for some} \quad e^2 = e \in P\}.$$

DEFINITION 1.1. Let S be an inverse semigroup and let G be a group. Then an E-unitary inverse semigroup P is an *E-unitary cover of S through G if*

(i) $P/\sigma \approx G$

(ii) there is an idempotent separating homomorphism θ of P onto S.