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UNMIXED 2-DIMENSIONAL LOCAL DOMAINS

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Let b, c be a system of parameters in a 2–dimensional local (Noetherian) domain (R, M). For $n \geqq 0$, the chain $(b^n: 1) \subset (b^n: c) \subset (b^n: c^2) \subset \cdots$ becomes stable. Thus define a function $S(b, c, -)$ by letting $S(b, c, n)$ be the least integer $k \geqq 0$ such that $(b^n: c^k) = (b^n: c^{k+1})$. Ratliff has shown that R is unmixed if and only if $S(b, c, -)$ is bounded. This paper shows that if R is unmixed then for any $0 \neq d \in M$ there is an integer $*d \geqq 0$ such that for any system of parameters b, c and any $i \geqq 0$, $S(b, c, *b + i) = *c$.

Introduction. We consider a 2–dimensional local domain (R, M) with a system of parameters b, c. (That is, b and c are nonzero nonunits, and no height 1 prime contains both of them.) For a fixed $n \geqq 0$, obviously $(b^n: 1) \subset (b^n: c) \subset (b^n: c^2) \subset \cdots$. As this chain eventually becomes stable, we define a function $S(b, c, -)$ by letting $S(b, c, n)$ be the least integer $k \geqq 0$ such that $(b^n: c^k) = (b^n: c^{k+1}) = \cdots$. A recent result of Ratliff shows that R is unmixed if and only if $S(b, c, -)$ is bounded. In this paper we show that if R is unmixed, then for any $0 \neq d \in M$ there is an integer $*d \geqq 0$ such that for any system of parameters b, c, and for any $i \geqq 0$, $S(b, c, *b + i) = *c$.

NOTATION. Throughout this paper, (R, M) will be a 2–dimensional local domain and b, c will be a system of parameters for R. For $d \in R$, $d^0 = 1$.

We consider the following two arrays of ideals, the displayed inclusions being trivial.

$$
\begin{array}{ccccccc}
(1: 1) & \subset & (1: c) & \subset & (1: c^2) & \subset & \cdots \\
\cup & & \cup & & \cup & & \\
(b : 1) & \subset & (b : c) & \subset & (b : c^2) & \subset & \cdots \\
\cup & & \cup & & \cup & & \\
(b^2 : 1) & \subset & (b^2 : c) & \subset & (b^2 : c^2) & \subset & \cdots \\
\cup & & \cup & & \cup & & \\
(b^3 : 1) & \subset & (b^3 : c) & \subset & (b^3 : c^2) & \subset & \cdots \\
\cup & & \cup & & \cup & & \\
\vdots & & \vdots & & \vdots & &
\end{array}
$$