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A PERTURBATION THEOREM FOR SPECTRAL OPERATORS

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This note is concerned with analytic perturbations of spectral operators. It is shown that under small perturbations simple isolated eigenvalues remain simple and isolated and depend holomorphically upon the perturbation parameter. As one would expect the bounds are rather complicated in the case of a spectral operator with general quasinilpotent part. For scalar operators, however, these bounds become simple and reproduce in the selfadjoint case those given by F. W. Schäfke.

For the sake of simplicity we deal only with bounded operators. The method used is an appropriate modification of the elegant Hilbert space method introduced by Schäfke in **[3]** to settle the analogous problem for selfadjoint operators. The generalization to the unbounded case is then straightforward.

Let X be a Banach space over $\mathbf{C}$ and $B(X)$ the algebra of bounded linear operators on X with the norm topology. Let further $\sigma(T)$, $\rho(T)$, and $R_\lambda(T) := (T - \lambda I)^{-1}$ for $\lambda \in \rho(T)$ denote the spectrum, the resolvent set, and the resolvent operator for $T \in B(X)$. Spectral measures, spectral operators and scalar spectral operators are defined as in Dunford–Schwartz **[1]**. Especially, if $S \in B(X)$ is a scalar spectral operator, we have

$$S = \int_{\mathbf{C}} \lambda E(d\lambda)$$

with an uniquely determined spectral measure E. A spectral operator can be uniquely decomposed as $T = S + N$, where S is a scalar spectral operator, and N is a bounded quasinilpotent operator commuting with the spectral measure E of S. If E is a spectral measure we denote by $\omega(E)$ the minimum of all reals c which obey

$$\left\| \int_{\mathbf{C}} f(\lambda) E(d\lambda) \right\| \leqq c \cdot \sup_{\lambda \in \mathbf{C}} |f(\lambda)|$$

for all bounded, Borel measurable, $\mathbf{C}$-valued functions on $\mathbf{C}$. Then $\omega(E) \geqq 1$.

If $T = S + N$ as above is a spectral operator we have