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ON RAMSEY THEORY AND GRAPHICAL PARAMETERS

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A graph G is said to have a factorization into the subgraphs $G_1, \cdots, G_k$ if the subgraphs are spanning, pairwise edge-disjoint, and the union of their edge sets equals the edge set of G. For a graphical parameter f and positive integers $n_1, n_2, \cdots, n_k$ $(k \geqq 1)$, the f-Ramsey number $f(n_1, n_2, \cdots, n_k)$ is the least positive integer p such that for any factorization $K_p = \bigcup_{i=1}^k G_i$, it follows that $f(G_i) \geqq n_i$ for at least one i, $1 \leqq i \leqq k$. In the following, we present two results involving f-Ramsey numbers which hold for various vertex and edge partition parameters, respectively. It is then shown that the concept of f-Ramsey number can be generalized to more than one vertex partition parameter, more than one edge partition parameter, and combinations of vertex and edge partition parameters. Formulas are presented for these generalized f-Ramsey numbers and specific illustrations are given.

1. Introduction. A subgraph H of a graph G is called *spanning* if H has the same vertex set as G. A graph G is said to have a *factorization* into the subgraphs $G_1, G_2, \cdots, G_k$, written $G = \bigcup_{i=1}^k G_i$, if the subgraphs are spanning, pairwise edge-disjoint, and the union of their edge sets equals the edge set of G. It is permissible for a subgraph G_i to be empty; i.e., have no edges.

Let f be a graphical parameter, and let $n_1, n_2, \cdots, n_k$, $(k \geqq 1)$ be positive integers. In [**2**], Chartrand and Polimeni defined the *f-Ramsey number* $f(n_1, n_2, \cdots, n_k)$ as the least positive integer p such that for any factorization $K_p = \bigcup_{i=1}^k G_i$ of the complete graph of order p, it follows that $f(G_i) \geqq n_i$ for at least one subgraph G_i, $1 \leqq i \leqq k$. If $\omega(G)$ is the maximum order among the complete subgraphs of G, then the ω-Ramsey number is the ordinary Ramsey number (see [**3**; p. 16]) in k variables.

The chromatic number $\chi(G)$ of a graph G is the minimum number of colors which may be assigned to the vertices of G so that adjacent vertices are assigned different colors. The vertex-arboricity $a(G)$ of G is the minimum number of subsets into which the vertex set of G may be partitioned so that each subset induces an acyclic subgraph. Chartrand and Polimeni [**2**] gave formulas for the χ-Ramsey numbers and the a-Ramsey numbers. We present a result which holds for several "partition" parameters (including chromatic number and vertex-