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A RANDOM FIXED POINT THEOREM FOR A MULTIVALUED CONTRACTION MAPPING

SHIGERU ITOH

Some results on measurability of multivalued mappings are given. Then using them, the following random fixed point theorem is proved: Theorem. Let X be a Polish space, $(T, \mathscr{A})$ a measurable space. Let $F\colon T \times X \to CB(X)$ be a mapping such that for each $x \in X$, $F(\cdot, x)$ is measurable and for each $t \in T$, $F(t, \cdot)$ is $k(t)$-contraction, where $k\colon T \to [0, 1)$ is measurable. Then there exists a measurable mapping $u\colon T \to X$ such that for every $t \in T$, $u(t) \in F(t, u(t))$.

1. Introduction. Random fixed point theorems for contraction mappings in Polish spaces were proved by Špaček [8], Hanš [2, 3], etc. For a brief survey of them and related results, we refer the reader to Bharucha–Reid [1, Chapter 3]. On the other hand, fixed point theorems for multivalued contraction mappings in complete metric spaces were obtained by Nadler [7], etc.

In this paper, in §3 we give some results on measurability and measurable selectors of multivalued mappings. Then in §4, using them we prove a random fixed point theorem for a multivalued contraction mapping in a Polish space.

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2. Preliminaries. Throughout this paper, let (X, d) be a Polish space, i.e., a separable complete metric space, and $(T, \mathscr{A})$ a measurable space. For any $x \in X$, $B \subset X$, we denote $d(x, B) = \inf\{d(x, y)\colon y \in B\}$. Let 2^X be the family of all subsets of X, $CB(X)$ the family of all nonempty bounded closed subsets of X, $\mathscr{B}$ the σ-algebra of Borel subsets of X, respectively. Let D be the Hausdorff metric on $CB(X)$ induced by d. A mapping $S\colon X \to CB(X)$ is called *k-Lipschitz*, where $k \geqq 0$, if for every $x, y \in X$, $D(S(x), S(y)) \leqq kd(x, y)$. When $k < 1$, then S is called *k-contraction*. A mapping $F\colon T \to 2^X$ is called *($\mathscr{A}$-)measurable* if for any open subset B of X, $F^{-1}(B) \in \mathscr{A}$, where $F^{-1}(B) = \{t \in T\colon F(t) \cap B \neq \varnothing\}$. Notice that in Himmelberg [5] this is called weakly measurable, but in this paper we use only this type of measurability for multivalued mappings, hence we omit the term 'weakly' for the sake of simplicity. A mapping $u\colon T \to X$ is said to be a *measurable selector* of a measurable mapping $F\colon T \to 2^X$ if u is measurable and for any $t \in T$, $u(t) \in F(t)$.