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COMPLETELY SEMISIMPLE INVERSE Δ-SEMIGROUPS ADMITTING PRINCIPAL SERIES

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A Δ-semigroup is a semigroup whose lattice of congruences is a chain with respect to inclusion. A completely semisimple inverse Δ-semigroup that admits a principal series is characterized here as a semigroup that results from a particular series of ideal extensions of Brandt semigroups by Brandt semigroups. A characterization is given of finite inverse Δ-semigroups in terms of groups, Brandt semigroups, and one to one partial transformations of sets.

1. Introduction. A *Δ-semigroup* is a semigroup whose lattice of congruences is a chain with respect to inclusion. Schein **[8]** and Tamura **[11]** showed that a commutative Δ-semigroup is either a quasicyclic group A, or a commutative nil semigroup B with the divisibility chain condition, or A^0, or B^1. We study here the structure of completely semisimple inverse Δ-semigroups with principal series. Such semigroups will be characterized in terms of Δ-groups, idempotent properties, and ideal extensions of Brandt semigroups by Brandt semigroups.

In **[11]** it was shown that the least semilattice congruence on a Δ-semigroup has at most two classes. We begin by characterizing completely semisimple inverse semigroups admitting principal series and having this property.

In the final section we show that each finite inverse Δ-semigroup determines a set of structure data that involves groups, Brandt semigroups and one to one partial transformations of sets. Conversely the semigroup can be reconstructed from the structure data.

2. Preliminaries. We call a semigroup S an $\mathscr{S}_1$-, or $\mathscr{S}_2$-*semigroup* if the smallest semilattice congruence on S has one, or two congruence classes respectively. S is a Δ-semigroup only if it is an $\mathscr{S}_1$- or $\mathscr{S}_2$-semigroup. In this section we characterize completely semisimple inverse $\mathscr{S}_1$-, or $\mathscr{S}_2$-semigroups that admit principal series.

A subsemigroup H of a semigroup S is $\mathscr{S}$-*unitary* if and only if whenever $HxyH \subseteq H$ for $x, y \in S^1$ then $Hx, yH \subseteq H$. Notice that if E is a semilattice and $efg = e$ in E then $ef = e = ge$. Hence, any class of a semilattice congruence on S is $\mathscr{S}$-unitary. Let $\mathscr{J}^*$ denote the least congruence on S containing Green's relation $\mathscr{J}$. For $a \in S$ let J_a be the $\mathscr{J}$-class of a and $J(a) = S^1aS^1$.