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SYMMETRIZABLE-CLOSED SPACES

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Symmetrizable-closed, semimetrizable-closed, minimal symmetrizable, and minimal semimetrizable spaces are characterized. G. M. Reed's theorem that every Moore-closed space is separable is extended to: Every Baire, semimetrizable-closed space is separable. Several examples are given.

If P is a topological property, a Hausdorff P-space will be called *P-closed* provided that it is a closed subset of every Hausdorff P-space in which it can be embedded. A Hausdorff P-space $(X, \mathcal{T})$ will be called *minimal P* if there exists no Hausdorff P-topology on X strictly weaker than $\mathcal{T}$.

In **[3]** J. W. Green characterized and studied Moore-closed and minimal Moore spaces. In this paper we obtain some analogous results for semimetrizable spaces and symmetrizable spaces.

A *symmetric* for a topological space X is a mapping $d: X \times X \to [0, \infty)$ such that

(1) For all $x, y \in X$, $d(x, y) = d(y, x)$, and $d(x, y) = 0$ if and only if $x = y$.

(2) A set $V \subset X$ is open if and only if for each $x \in V$ there exists $n \in \boldsymbol{N}$ such that V contains the set $B(n, x) = \{y \in X \mid d(x, y) < 1/n\}$.

A space X which admits a symmetric is said to be *symmetrizable*, and if, in addition, each $B(n, x)$ is a neighborhood of x, then X is said to be *semimetrizable* and d is called a *semimetric* for X. Equivalently, X is semimetrizable via d provided that for $x \in X$, $A \subset X$, and $d(x, A) = \inf\{d(x, a) \mid a \in A\}$, the condition $x \in \bar{A}$ if and only if $d(x, A) = 0$ is satisfied.

A number of the techniques used here are not new; for example, see **[2]**. The terminology used is standard. One perhaps not too familiar concept is that of θ-adherence. A point p of a topological space is said to be a *θ-adherent point* (or be in the *θ-adherence*) of a filter base $\mathcal{F}$ provided that for every set $F \in \mathcal{F}$ and neighborhood V of p, one has $F \cap \bar{V} \neq \emptyset$.

Our first two theorems are characterization theorems.

THEOREM 1. *Let $(X, \mathcal{T})$ be a symmetrizable Hausdorff space. The following are equivalent.*

(i) *The space $(X, \mathcal{T})$ is minimal symmetrizable.*

(ii) *Every countable filter base on $(X, \mathcal{T})$ which has a unique θ-adherent point is convergent.*