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ON COMPLETENESS OF THE BERGMAN METRIC AND ITS SUBORDINATE METRICS, II

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Let M be a complex manifold of dimension n furnished with both the Bergman metric and the Carathéodory distance. The main result of the present paper is to prove that the Bergman metric is always greater than or equal to the Carathéodory distance on M. The case where M is a bounded domain in the space C^n was already considered by the author in Proc. Nat. Acad. Sci. (U.S.A.), 73 (1976), 4294.

1. Introduction. The main purpose of the present paper is to prove the following

THEOREM A. *Let M be a complex manifold which admits both the Bergman metric s_M and the Carathéodory differential metric α_M. For each $z \in M$ and each holomorphic tangent vector ξ,*

$$\alpha_M(z, \xi) \leqq s_M(z, \xi). \tag{1}$$

Let ρ_M and d_M denote the integrated metrics on M which are induced from α_M and s_M, respectively. Then the Carathéodory distance c_M ([2]) satisfies

$$c_M \leqq \rho_M \leqq d_M \tag{2}$$

and there are cases when ρ_M differs from c_M and d_M.

From this observation and Theorem A, we obtain

THEOREM B. *Let M be a complex manifold given as in Theorem A. Then the Bergman metric is complete in M whenever the Carathéodory distance is complete.*

If in particular M is a bounded domain in the complex Euclidean space C^n $(n \geqq 1)$, M always admits the Bergman metric and the Carathéodory differential metric.

Theorems A and B have a number of interesting consequences.

In [4], C. Earle has proved the completeness of the Carathéodory distance in the Teichmüller space $T(g)$ of a compact Riemann surface of genus $g \geqq 2$. Therefore, Theorem B immediately implies the following