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S-SPACES IN COUNTABLY COMPACT SPACES USING OSTASZEWSKI'S METHOD

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A method adapted from that used by A. J. Ostaszewski is used to construct S-spaces as subspaces of given spaces. Assuming the set-theoretic principle $\diamondsuit$, it is shown that every countably compact space containing no nontrivial convergent sequences contains a perfect S-space. As a corollary, assuming $\diamondsuit$, if X is a countably compact F-space, then X contains a hereditarily extremally disconnected, hereditarily normal, perfect S-space.

1. Introduction. The set-theoretic principle $\diamondsuit$, due to Jensen **[3]**, has found many interesting applications in topology, particularly the construction of Souslin lines and various S-spaces. The basic technique for constructing S-spaces from $\diamondsuit$ is due to A. J. Ostaszewski **[6]**, and has been modified and applied in constructing other interesting topological spaces, notably in **[5]** and **[8]**. Roughly speaking, the method involves constructing a space having desired properties by defining its topology inductively over more and more of the space (and in some cases refining a given topology) using some principle of enumeration.

Here we will show how the method can be used to construct S-spaces as subspaces of given spaces. That is, rather than building up a space by inductively defining its topology, the desired examples will be obtained by working within a given topological space and extracting a subspace.

Our principal topological references are **[2]**, **[7]** and **[10]**. For set-theoretic notions we refer to **[4]**.

For the reader's convenience we now recall a few notions from topology which we will employ.

A space X is an *S-space* if X is regular, hereditarily separable and not Lindelöf.

X is *countably compact* if every countable covering of X by open sets has a finite subcover.

For a completely regular space X, βX denotes the Stone–Čech compactification of X.

A subset A of X is *C^*-embedded* in X if every bounded, continuous real-valued function on A admits a continuous extension to X. A *cozero-set* in X is a set of the form $\{p \in X : f(p) \neq 0\}$ where f is a continuous real-valued function on X. X is an *F-space* if X is com-