ON NONSINGULARLY k-PRIMITIVE RINGS

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A ring R is called k-primitive if it has a faithful cyclic critical right module C with $|C| = k$. We first show that k-primitive rings with Krull dimension have many properties in common with prime rings. For the case where R is a PWD with a faithful critical right ideal, we obtain an internal characterization.

1. Introduction. Let R be a ring with Krull dimension. Then R is prime if and only if R has a faithful compressible right R-module. In this paper we consider a broader class of rings, those which have a faithful cyclic critical right R-module. From [2] such a ring is called a *k-primitive* ring where k denotes the Krull dimension of the faithful critical.

In the case where the faithful critical is nonsingular, these rings exhibit many of the properties of prime rings. Not all k-primitive rings have this additional property as an example in §4 shows. We call a k-primitive ring whose faithful critical is nonsingular, a *nonsingularly k-primitive* ring. Section 2 is devoted to showing some of the similarities with prime rings.

In §3 we consider piecewise domains (PWD) which are k-primitive rings. An internal characterization of PWD's with faithful critical right ideal is obtained, which is our main result.

All rings will have identity, and the modules are right unital. The singular submodule of a module M_R is denoted $Z(M)$. If X is a subset of R, then ann X or X^r denotes the right annihilator of X in R. The Krull dimension of a module M_R is denoted by $|M|$. A certain familiarity with the definitions and basic results concerning Krull dimension is assumed. See [5] for reference.

2. Properties of k-primitive rings. If R is a prime ring with Krull dimension then R is nonsingular and has a faithful critical C such that $|C| = |R|$. These conditions are also true for nonsingularly k-primitive rings.

PROPOSITION 2.1. *Let R be a k-primitive ring with faithful cyclic critical C. Then $Z(R) = 0$ and $|C| = |R|$ if and only if R is nonsingularly k-primitive.*