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PAIRS OF SYMMETRIC BILINEAR FORMS IN CHARACTERISTIC 2

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The Grothendieck group of finite-length inner product modules over a PID is here shown to be a sum of countably many copies of the corresponding groups for the residue fields. It follows that nonsingular pairs of symmetric bilinear forms in characteristic 2 owe their extra complexity only to lack of a cancellation theorem: The invariants for isometry in other characteristics continue to determine classes in the Grothendieck group. This is also true for singular pairs.

1. Inner products on finite-length modules. Let R be a principal ideal domain, K its fraction field, and E the R-module K/R. If M is a finite-length R-module, then $M^* = \mathrm{Hom}_R(M, E)$ is abstractly isomorphic to M, and the canonical map $M \rightarrow (M^*)^*$ is an isomorphism [1, p. 94-97]. An R-bilinear function $B: M \times M \rightarrow E$ induces a homomorphism $m \mapsto B(m, -)$ from M to M^*, and every such homomorphism arises from a unique B. Since M and M^* have the same length, B is nondegenerate iff $M \rightarrow M^*$ is an isomorphism. Identifying M^{**} canonically with M, we see that B is symmetric iff $M \rightarrow M^*$ is self-adjoint. A nondegenerate symmetric B is called an *inner product* on M.

Clearly the (orthogonal) direct sum of two modules with inner products is again one. The set of isometry classes is thus a commutative semigroup, and we can form the associated Grothendieck group. If 2 is invertible in R, Theorem 1.3 of [6] implies that the semigroup has a cancellation theorem, so passage to the Grothendieck group changes nothing. In residue characteristic 2, the isometry classes are more complicated, but it turns out that the Grothendieck group still has the same structure:

THEOREM 1. *Let R be a principal ideal domain, and suppose that a specific generator p has been chosen for each prime ideal pR. The Grothendieck group of inner products on finite-length R-modules is then canonically isomorphic to*

$$\bigoplus_p \bigoplus_{n=1}^{\infty} WG(R/pR)\ ,$$

where $WG(R/pR)$ is the Grothendieck group of inner product spaces over R/pR.