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EQUATIONS OF MEAN CURVATURE TYPE IN 2 INDEPENDENT VARIABLES

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The object of this paper is to develop a regularity theory for equations of mean curvature type in two independent variables. An equation of mean curvature type in two independent variables is defined to be an equation of the form

$$\sum_{i,j=1}^{2} a_{ij}(x, u, Du)D_{ij}u = b(x, u, Du)$$

on a domain $\Omega \subset \boldsymbol{R}^2$, where the functions a_{ij}, b satisfy special structural conditions. Namely, we require that (i) $(1 + |Du|^2)^{-1/2}b(x, u, Du)$ is bounded by a fixed constant (independent of u), and (ii) the quadratic form $\sum_{i,j=1}^{2} a_{ij}(x, u, Du)\xi_i\xi_j$ is bounded from above and below in terms of the quadratic form $\sum_{i,j=1}^{2} g^{ij}(Du)\xi_i\xi_j$, where $g^{ij}(Du) = \delta_{ij} - D_iuD_ju/(1+|Du|^2)$, $i, j = 1, 2$, are the coefficients of the minimal surface equation.

R. Finn [2] was the first to consider such equations; he considered the case $b \equiv 0$ and $a_{ij}(x, u, Du) \equiv a_{ij}(Du)$. Later Jenkins [5] and Jenkins-Serrin [6] specialized further to equations which arise as the non-parametric Euler-Lagrange equation of a parametric elliptic functional with integrand independent of the spatial variables (see Appendix 1). The main results in [2] concerned a-priori estimates for the gradient of a solution. In [5], [6] somewhat deeper results were obtained; in particular, pointwise estimates for the principal curvatures of the graph of a solution were established. Recently J. Spruck [11] obtained such a pointwise curvature estimate for the constant mean curvature equation; this was the first such result obtained for a non-homogeneous (i.e. b not identically zero) equation of mean curvature tpye.

In this paper we intend to use the Hölder estimate established in [8] in order to obtain a strong regularity theory for the entire class of equations of mean curvature type. The plan of the paper is as follows. In §1 we introduce the class of equations of mean curvature type and give a geometric characterization of such equations. In §2 we discuss application of the results of [8] to *homogeneous* equations of mean curvature type; in particular we obtain some a-priori gradient estimates, a Bernstein type theorem, a Bers-type theorem concerning the limiting behaviour of the gradient of solutions defined outside a compact set, a global Hölder continuity estimate for solutions which continuously attain Lipschitz boundary