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NONCOMPACT, MINIMAL REGULAR SPACES

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In the first known example of a noncompact, minimal regular space, the standard Tychonoff plank is used in the construction. In the present paper, this construction is generalized by using, instead of the standard Tychonoff plank, a generalized Tychonoff plank which is defined in terms of an arbitrary pair of ordinals. Such a construction yields a noncompact, minimal regular space if and only if both ordinals are limit ordinals and at least one of them is uncountably cofinal.

1. Introduction. Throughout this paper, ω and Ω shall denote, respectively, the first infinite ordinal and the first uncountable ordinal. For an arbitrary ordinal α, we shall let α' denote the set of all ordinals less than or equal to α, with the order topology.

In [2], Berri and Sorgenfrey used countably infinitely many copies of the deleted Tychonoff plank $\omega' \times \Omega' - \{(\omega, \Omega)\}$ to construct a noncompact, minimal regular space. A natural speculation might be that the same construction could be used with any ordinals in place of ω and Ω, or possibly any limit ordinals or any two limit ordinals of different cardinality. However, it will be shown here that such a construction yields a noncompact, minimal regular space if and only if both ordinals are limit ordinals and at least one of them is uncountably cofinal.

It is noted here that other techniques of construction—for example, extension spaces—can be used to obtain noncompact, minimal regular spaces. For a discussion of such techniques, the reader is referred to [1], [3], [5], [7] and [9]. For further discussion of results concerning limit ordinal numbers, the reader is referred to [4], [6] and [8].

Finally, it is noted that the principal result of this paper was obtained simultaneously by the first two authors, working together, and the third author, working independently.

2. The generalized Tychonoff plank. The Tychonoff plank $\omega' \times \Omega'$ has been the source of many counter-examples in general topology. For example, the Tychonoff plank with the point (ω, Ω) removed is completely regular but not normal. In this section, we shall define generalized Tychonoff planks and prove a result, concerning these spaces, which will be used in § 3.