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SEMIGROUPS WITH IDENTITY ON PEANO CONTINUA

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A continuum is cell-cyclic if every cyclic element is a finite dimensional cell. We show that any finite dimensional cell-cyclic Peano continuum X admits a commutative semigroup with zero and identity, and apply this to show that if X is also homogeneous it is a point.

In [12] we showed that each cell-cyclic Peano continuum (locally connected metric continuum every cyclic element of which is a finite dimensional cell) X admits a semilattice (commutative idempotent topological semigroup). We now extend this result to show that X admits a commutative semigroup with identity and zero, and then apply this to homogeneous continua. Our extension is a partial answer to a question first raised by R. J. Koch in [6].

A semilattice is also a partially ordered Hausdorff topological space in which every two elements have a greatest lower bound and the function $(x, y) \rightarrow glb\{x, y\}$ is continuous. For $A \subset S$, let $L(A) = \{z: z \leqq x$ for some $x \in A\}$ and $M(A) = \{y: x \leqq y$ for some $x \in A\}$. A set A is *increasing* if $M(A) = A$. An *arc chain* is a totally ordered subset of a semilattice whose underlying space is an arc. We reserve I for the unit interval under min multiplication, and T for the quotient semilattice obtained by identifying (0, 0) and (1, 0) in $\{0, 1\} \times I$. Note that I^n and T^n, under coordinatewise multiplication, are semilattices with identity on the n-cell, with zero in the boundary and interior respectively.

Let X be a cell-cyclic Peano continuum. We use the cyclic element notation and results of Whyburn [10] and Kuratowski and Whyburn [8], slightly modified in the following way. In X we say a set A separates a and b if each arc from a to b meets A. $C(p, q)$ denotes the *cyclic chain from p to q* and is $\{x \in X |$ some arc from p to q contains $x\}$. An subcontinuum A of X is an *A-set* if each arc in X having end points in A is contained in A. Cyclic elements and cyclic chains are A-sets. Given a point x and an A-set A, if $x \notin A$ there is a unique element $y \in A$ such that y separates each element of A from x. Denote this y by $P(A, x)$. If $x \in A$ set $P(A, x) = x$. Then for a fixed A-set A the function $x \rightarrow P(A, x)$ is a monotone retraction of X onto A mapping $X \backslash A$ into $Fr(A) = \{x \in A | x \notin D^0$ for any cyclic element D of $A\} \cup \{$cut points of $A\}$. A set M is *nodal* in X if $M \cap (X \backslash M)^*$ contains at most one point. A point is an *end point* of X if it has a basis of neighborhoods having one point