PACIFIC JOURNAL OF MATHEMATICS
Vol. 69, No. 2, 1977

CLASSES OF RINGS TORSION-FREE OVER THEIR CENTERS

LOUIS ROWEN

Let $J(\ )$ denote the intersection of the maximals ideals of a ring. The following properties are studied, for a ring R torsion-free over its center C: (i)$J(R) \cap C = J(C)$; (ii) "Going up" from prime ideals of C to prime ideals of R; (iii) If M is a maximal ideal of R then $M \cap C$ is a maximal ideal of C; (iv) if M is a maximal (resp. prime) ideal of C, then $M = MR \cap C$. Properties (i)-(iv) are known to hold for many classes of rings, including rings integral over their centers or finite modules over their centers. However, using an idea of Cauchon, we show that each of (i)-(iv) has a counterexample in the class of prime Noetherian PI-rings.

Let R be a ring with center C. Throughout this note, we assume that R *is torsion-free as C-module*, i.e., $rc \neq 0$ for all nonzero r in R, c in C. (In particular, this is the case if R is prime.) Let $J(R) = \cap$ {maximal ideals of R}.

R is a *PI-ring* if there exists a noncommutative polynomial $f(X_1, \cdots, X_m)$ with coefficients ± 1, such that $f(r_1, \cdots, r_m) = 0$ for all r_i in R. The basic facts about PI-rings are in [6, Chapter X], as well as in [10]. Kaplansky's theorem implies that if R is a PI-ring, then $J(R)$ is the Jacobson radical of R, so clearly $J(R) \cap C \subseteq J(C)$. A natural question is, "Under what conditions does $J(R) \cap C = J(C)$?," or, more generally, "Is there any general correspondence between $J(R)$ and $J(C)$?" An answer for PI-rings given in [12, Theorem 5.9], is that $J(R) = 0$ implies $J(C) = 0$. The object of this note is to tie this question in with other notions which often arise (especially in PI-theory). Then we give some pathological examples, which show that many interesting negative properties (including $J(R) \cap C \neq J(C)$) occur in such natural classes as the class of prime Noetherian PI-rings. Some easy theory is developed to cast some light on the sharpness of these counterexamples. (Although the counterexamples are associative, one may note that associativity is not needed in the positive results.)

Call an ideal A of C *contracted* if $A = A' \cap C$ for some ideal A' of R. (By [11, Theorem 2], semiprime PI-rings have a wealth of contracted ideals of the center.)

LEMMA 1. *An ideal A of C is contracted, iff $AR \cap C \subseteq A$.*

Proof. Suppose A is contracted, i.e. $A = A' \cap C$. Then $AR \subseteq A'$,