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COMPLEMENTATION IN THE LATTICE OF CONVERGENCE STRUCTURES

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This paper classifies the various complements in the lattice of convergence structures and considers some of the properties shared by given structures and their relative complements or differences.

1. Introduction. The lattice of all convergence structures on a nonempty set has been investigated by several authors with some consideration given to the smaller lattices of pseudotopologies and pretopologies. The purpose here will be to show the lattice of convergence structures is a Stone lattice and has pseudo-differences, completely characterize relative pseudo-complements and pseudo-differences, establish that some types of convergence structures retain their classification after finding relative complements or differences and exhibit certain standard lattice operators as homomorphisms.

Lattice definitions follow those of Rasiowa and Sikorski [11], Birkhoff [2], and Szasz [15]. An element x of a lattice L is *compact* if whenever $x \leqq \vee\{y_j \mid j \in J\}$ implies that $x \leqq \vee\{y_j \mid j \in K\}$ for some finite subset K of J. The lattice L is *compactly generated* if each element of L is the join of compact elements. *Cocompact* elements and *cocompactly generated* lattices are defined dually. An element z in L is the *pseudo-complement of x relative to y* ($x*y$) if z is the greatest element such that $x \wedge z \leqq y$. If L has a least element 0, the *pseudo-complement* $-x$ of x is the greatest element for which $x \wedge (-x) = 0$. A *Brouwerian lattice* is one in which the relative pseudo-complement of any two members always exists; a Brouwerian lattice with a least element is *pseudo-Boolean* and a pseudo-Boolean lattice is a *Stone lattice* if there exists a greatest element 1 and $(-x)\vee(--x) = 1$ for every x in L. An element z is the *pseudo-difference* of y and x ($y - x$) if z is the least element such that $y \leqq x \vee z$.

Definitions on convergence structures used herein can be found in [1], [7] or [9]. $C(X)$ will denote the family of all convergence structures on a nondegenerate set X, $F(X)$ the lattice of filters on X and $U(X)$ the set of ultrafilters. In this paper the power set or improper filter is considered a member of $F(X)$. This makes $F(X)$ a complete lattice and facilitates the description of pseudo-differences in terms of the filter quotients of Ward [16].

For a point x, $\dot{x}$ will be the principal filter on X generated by $\{x\}$ while in $F(X)$ $[\mathscr{F}]$ signifies the principal filter generated by a